Dynamic Relational Contracts under Limited Liability

Jonathan P. Thomas[1] and Tim Worrall[2]

[1]Management School and Economics
University of Edinburgh

[2]Centre for Economic Research
Keele University

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Introduction

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- Contracts cannot be enforced and in the event of disagreement agent i receives a breakdown payoff of $\phi_i(a_1, a_2, s)$.
- Agents are risk neutral and have limited liability.
- The agents interact repeatedly over an infinite horizon.

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- Examine constrained Pareto-efficient infinite horizon self-enforcing contract.
- Interested is large but not too large discount factors.
- We want to know:
  - What actions are chosen and how do they vary over time?
  - How is the surplus divided and how does this change over time?

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- Repeated Credit Relationship
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- Repeated Employment Relationship
  - Employer provides tools t and worker hours h to produce project output $y(h, t)$.

What to Expect

- There is a hold-up problem so we should expect some under-investment.
- The backloading principle which says that best way to prevent an agent from reneging is to backload transfers into the future which would be forgone if the agent reneged.
  - But the principle applies in the one-sided case where one side can commit.
- The results of Ray (2002) and others suggest convergence to a stationary phase.
  - But again only in the one-sided case. Our environment is much more general.

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- There is convergence to a stationary phase despite a general stochastic structure.
- There is surplus maximization in the stationary phase.
- There is monotonic convergence in some cases (the additive case).
- We don't use dynamic programming (because of problems of non-convexity and non-differentiability).

Technology

- An infinite sequence of dates, $t = 1, 2, 3, \ldots,$
- There is a finite set of states which follow a time homogeneous Markov process with transition matrix Π with elements π_{sr}.
- The state at date t is denoted s_t and the history of states up to date t is denoted $s^t = (s_0, s_1, \ldots, s_t)$.
- After the event s^t is observed, both agents choose an action $a_i \in A_i(s_t)$
- These actions produce a nonnegative output $y(a_1, a_2, s_t)$.
- The function $y(\cdot, \cdot, s_t)$ is twice continuously differentiable, strictly increasing and strictly concave (where positive) in (a_1, a_2) and has strategic complementarity. $a_1^*(a_2)$ and $a_2^*(a_1)$ are the efficient solutions.

Preferences

- The consumption of agent i in event s^t is denoted $c_i(s^t)$.
- We shall assume that this consumption must be non-negative and feasible:

$$\sum_{i=1}^{2} c_i(s^t) \leq y(a_1, a_2, s_t) \quad \forall s^t.$$

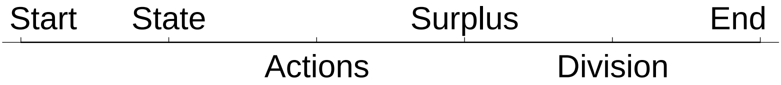
- Agents are risk-neutral and their per-period utility is:

$$w_i(s^t) = c_i(s^t) - a_i(s^t).$$

- Agents have a common discount factor $\delta \in (0, 1)$ and are interested in maximizing expected discounted utility

$$\mathrm{E}\left[\sum_{t=0}^{\infty} \delta^t w_i(s^t) \mid s_0\right]$$

Time Line



Deviations

Agents can deviate either at the action determination stage (after the state is realised) but before the surplus is known, or at the surplus division stage once the surplus is known (and the action of the other agent is observed).

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  - is strictly increasing in the action of the other agent.
  - is concave in the action of the other agent.
- The payoff along an equilibrium outcome path, $(a(s^t), w(s^t))_{t=1}^{\infty}$, for agent i is

$$V_i(s^t) \equiv w_i(s^t) + \mathrm{E}\left[\sum_{\tau=t+1}^{\infty} \delta^{\tau-t)} w_i(s^\tau) \mid s^t\right].$$

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- A necessary and sufficient condition for the outcome path $(a(s^t), w(s^t))_{t=0}^{\infty}$ to be an equilibrium is that it is feasible, and for every s^t

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- We refer to paths that satisfy (1) as self-enforcing and the inequalities themselves as the self-enforcing or incentive constraints.
- Whenever (1) holds with equality agent i is constrained otherwise we say that agent i is unconstrained.
- A self-enforcing agreement $\Gamma(s^t) = (a(s^t), w(s^t))_{t=0}^{\infty}$ is one that is both feasible and self-enforcing.

Breakdown Payoffs

- At the division stage agents play a Nash demand game in which both agents simultaneously announce utility claims (w_1, w_2).
  - If these claims are feasible, then this determines the split of the surplus.
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- The breakdown payoff for agent i in state s is $\phi_i(a_1, a_2, s_t) - a_i$ as a function of the actions taken by both agents. It is assumed the breakdown payoffs satisfy

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- In the breakdown stage game agents i chooses the best response $a_i^N(a_j, s^t)$ to maximize $\phi_i(a_1, a_2, s_t) - a_i$.

Nash Reversion

- With Nash reversion: after any deviation the game goes to breakdown in the current period and thereafter both agents play 'the' Nash equilibrium of the breakdown game.
- Then $D_i(a, s_t)$ denote the best non-cooperative discounted payoff for i starting from action a in state s, given Nash reversion.
- This is given by

$$D_i(a_j, s_t) = \phi_i(a_i^N(a_j, s_t), a_j, s_t) - a_i^N(a_j, s_t) + \delta \sum_{s_{t+1} \in \mathcal{S}} \pi_{s_t s_{t+1}} D_i(a_j^{NE}(s_{t+1}), s_{t+1}).$$

Pareto-Efficient Self-enforcing Agreements

- Aim to characterise Pareto-efficient self-enforcing agreements.
- Let $\mathcal{G}_s$ be the set of self-enforcing contracts contracts.
- Associated with any $\Gamma(s^t) = \{a(s^\tau \mid s^t), w(s^\tau \mid s^t)\}_{\tau=t}^{\infty} \in \mathcal{G}_s$ is a pair of future utilities $(V_1(s^t), V_2(s^t))$.
- Let $\mathcal{V}_s$ denote this set.
- The set is closed and bounded and non-empty.
- We are interested the Pareto-frontier $\Lambda(\mathcal{V}_s)$.

Results: Efficiency

- If A is unconstrained then B doesn't under-invest.
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  - Reducing A's investment increases surplus but reduces output and so consumption must fall. This is possible if A's consumption is positive.
- Actions are above the Nash reaction functions.
  - Increasing A's action up to Nash level increases surplus but extra consumption has to be given to B to prevent him reneging. However, if below Nash level the increase in breakdown less than then increase in surplus $(\partial y/\partial a_1) - 1$ and so A's utility can also be increased.

Results: Backloading

- If A gets all the output, then A is unconstrained.
- If A is unconstrained and underinvesting then B has zero consumption at all previous dates — payments are optimally backloaded into the future.
- It may be worthwhile to backload utility by having one agent overinvest.

Results: Backloading

- If A gets all the output, then A is unconstrained.
  - A can't get any more in the current period by reneging and there will be a future penalty for reneging by construction.
- If A is unconstrained and underinvesting then B has zero consumption at all previous dates — payments are optimally backloaded into the future.
  - Otherwise increase A's action. This requires an increase in B's consumption today. Do this by reducing B's consumption yesterday and transferring to A to compensate for increased action today.
- It may be worthwhile to backload utility by having one agent overinvest.
  - This has a real cost so do only if consumption is already backloaded.

Results: Convergence

Theorem (Convergence)

For any optimum contract, there exists a random time T which is finite with probability one such that for $t \geq T$, a^t is current joint surplus maximizing. In any state $s_t \in \mathcal{S}$ in which full efficiency a^* is not achievable for any division of the surplus, then both self-enforcing constraints bind and there is under-investment by both agents.

If actions are positive at some point, then so too must be consumption of both agents. By backloading, there is no over-investment thereafter and then we can conclude there is either under-investment by both agents or efficiency is attainable. In either case this means joint utility maximization and hence joint surplus maximization at each subsequent date.

Two-phase Property

Where actions are positive, true if say, the Nash actions are positive, then we have the following two-phase property.

Theorem (Two-Phase Contract)

Whenever the Nash actions $a_{i,s}^{NE}$ *are positive,* $i = 1, 2$, *then with probability one an optimum path has two phases, where* $i = 1$ *or* 2*:*

Phase 1: $c_i^t = 0$, $a_i > a_i^*$ *and* $a_j \leq a_j^*$, *for* $0 \leq t < \tilde{t}$, $j \neq i$, *where* $\infty > \tilde{t} \geq 0$;

Phase 2: $a_1^t \leq a_1^*$ *and* $a_2^t \leq a_2^*$ *for* $t \geq \tilde{t}$ *and after the first period of phase 2, if* a^* *is feasible in* s_t *then* $a^t = a^*$; *otherwise* $a < a^*$, *both constraints bind, and* $c > 0$.

Numerical Example

- Similar to Garvey's (1995) model of joint ventures:
- Additive production, $y(a_1, a_2) = f_1(a_1) + f_2(a_2)$ and each agent can grab the other's output.

Simple Example without Uncertainty

The production functions are $f_1(a_1) = 2b\sqrt{a_1}$ and $f_2(a_2) = 2\sqrt{a_2}$ for a parameter $b \in (0,1)$. The breakdown game payoffs are $\phi_1(a_2) = 2\sqrt{a_2}$ and $\phi_2(a_1) = 2b\sqrt{a_1}$. We set $\delta = \frac{1}{3}$ and $b = \frac{\sqrt{3}}{3}$.

The Pareto-Frontier, Surplus and Actions

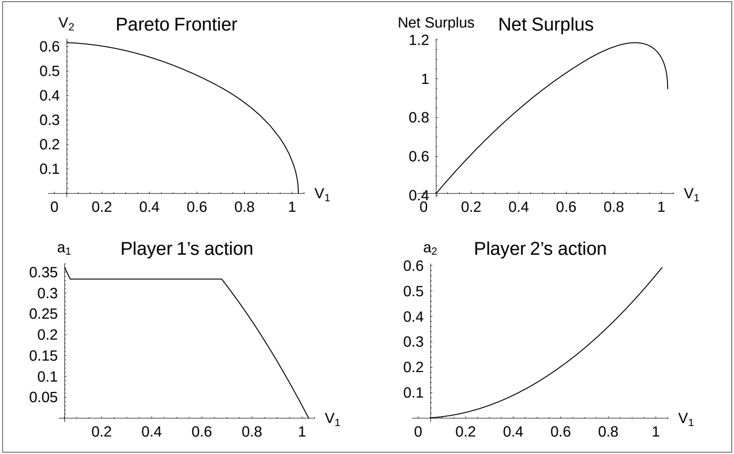

Figure: The Pareto-Frontier, Surplus and Actions

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Conclusions

- We have confirmed some of the results from the existing literature.
  - We confirm the result of Ray (2002) that after a finite time we move to a situation where the joint surplus is maximised over all possible self-enforcing agreements.
  - We confirm the backloading result that where an agent is constrained, payments to the agent are optimally backloaded into the future.
  - We confirm the result of Thomas-Worrall (1994) and Albuquerque-Hopenhayn (2004) that the actions of the unconstrained agent tend to gradually increase over time.
- We show that the backloading incentive is severe enough that the constrained agent might inefficiently overinvest.
- We show that at the stationary solution (if the efficient actions cannot be sustained) then both agents are constrained.