Cost Padding in Regulated Monopolies

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Abstract

This paper considers a regulated monopoly that can pad or falsify its costs to increase its cost reimbursement from a regulator. The firm can also engage in a cost reducing investment before it enters into a regulatory contract. The investment in cost reduction determines the firm type and the paper derives the optimum incentive compatible falsification contracts and an equilibrium for the type distribution. It shows that at the optimum price setting regulation is relaxed and the regulator tolerates some cost padding. There is under-investment in cost reduction and investment is distorted away from the cost minimizing level. It also shows that where there is an equilibrium type distribution it is continuous and there are no mass points.

Keywords: Cost padding; costly state falsification; endogenous screening.

JEL Codes: D82; L43; L52.

1. Introduction

It is a serious concern among regulatory agencies that firms engage in accounting contrivances and cost padding to increase their remuneration from the regulator.[1] Firms usually have many ways of diverting funds to raise or pad costs: increasing salaries and expense claims, "gold-plating" of expenditures, charging other equipment to project costs, advertising for corporate image, charging for depreciated assets, not reporting of cost reducing improvements and so on.

The extent of such cost padding is difficult to quantify as by definition it is hidden. However it can be substantial. The report by the Godbole committee into the Enron-owned Dabhol Power Company in Maharashtra State, India found cost padding of Rs 930 crore (about $200 million). Other evidence derives from legal action. In one case of the US government versus the defense contractor Sundstrand in 1989, a sum of $200 million

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[1]For examples of such reports related to public utilities, the reader is referred to McAfee and McMillan (1988, government contracting - North America), Quiggin (1998, electricity - Australia); Kerr (1998, water - New Zealand); Department of Transportation and Regional Services (2000, transport - Australia); Ontario Federation of Agriculture (1999, energy - Canada); Watson (2000, public utilities - Australia), and the OECD study by Gonenc, Maher, and Nicoletti (2000) that compares the incentives that price-cap regulation provides for cost-padding in electricity and telecommunications. For examples in procurement contracts, the reader is referred to Manoj (2000, Shipping - India) and Higgs (1998, military - U.S.A).

was recovered as the court found that Sundstrand had co-mingled commercial and government costs. In another case from 1985, a U.S. federal grand jury indicted the General Electric Company on charges that it had falsified claims for work on a nuclear warhead system. It was alleged that the government had been defrauded of at least $800,000 between January 1980 and April 1983 because the company had entered exaggerated charges on employee time cards.[2] Another piece of evidence comes from a 1984 audit by the U.S. Department of Defense inspector-general, which found that contractors were inflating charges for spare parts and tools in over one third of all cases.[3,4]

Whilst cost padding is an important practical consideration for regulators the main theoretical models of regulation in industrial economics are ill-suited to addressing this question.[5] In Baron and Myerson (1982) for example it is assumed that the regulator is totally unable to observe the firm's costs and therefore since the payment to the firm does not depend upon costs the firm has no incentive to pad them. Equally in the standard model of Laffont and Tirole (1993) it is assumed that the regulator observes the firm's costs perfectly but is unable to observe the firm's effort in cost reduction. Again the firm has no incentive to pad costs as the true cost is perfectly and costlessly monitored. This paper considers an intermediate case where the regulator can perfectly observe total cost, that is true cost plus padded costs but cannot disentangle the two components.

The approach we adopt to study cost padding is to turn the Laffont-Tirole model on its head. We allow for a difference between real and observed costs at the contracting stage and study the incentives for cost reduction by allowing the firm to undertake a cost-reducing investment at a pre-contractual stage.[6] Padded costs, which are the difference between real and observed costs, are treated as a post-contractual hidden action of the

[2]Another case from 1969 concerned the supply of rocket motor assemblies for the GENIE weapons system by McDonnell-Douglas to the US military. The armed services board of contract appeals found that McDonnell-Douglas had failed to disclose actual experienced manufacturing hours and failed to disclose information about inventories and the latest available prices and quotations on purchased parts. The board found that the government was entitled to a $54,235 price reduction.

[3]This overpricing amounted in total to only 6 per cent of the value of the equipment in question. Reports of contractors' overcharging the Pentagon however, appear regularly: thus it has been claimed that hammers selling for $7 in hardware stores were charged to the Pentagon at $436; that a small plastic cap worth 75 cents was charged at $1,118 per unit; that a 25-cent plastic washer was charged at $400 per unit. In 1984, the Air Force paid $7,600 for a coffee maker for use in a Lockheed transport aircraft. Although often cited these inflated prices are in part consequence of the Pentagon's accounting rules: a large proportion of the price of the $436 hammer consists of overhead and extra labor costs charged in accordance with Pentagon regulations (see McAfee and McMillan (1988)).

[4]Some indirect evidence of the potential scale of cost padding may be garnered from the study of hierarchical organizations by McAfee and McMillan (1995) who cite the studies by Berliner (1957), Schiff and Lewin (1968) and Schiff and Lewin (1970). Berliner interviewed former managers of Soviet firms and found as one manager reported "an enormous amount of falsification in all branches of production and in their accounting systems..." (p.161). Schiff and Lewin studied the efficiency of divisions within three large U.S. corporations and produced estimates of the size of cost-padding within divisions of the same company to be between 20 and 25%.

[5]For a survey of these studies see e.g. Laffont (1994).

[6]Levine and Rickman (2002) also consider a regulated firm that makes an ex ante non-contractible investment in cost reduction. They allow delegation to a regulatory agency to partially overcome the commitment problem.

firm to increase its cost reimbursement whereas the pre-contractual investment in cost reduction is treated as the hidden information of the firm at the contracting stage. In treating cost padding as a hidden action of the firm we follow the literature on costly-state falsification initially proposed and analyzed by Lacker and Weinberg (1989).[7] In determining the investment in cost reduction at the pre-contractual stage we follow the approach of González (2004) and Gul (2001) on endogenous screening and derive the distribution of cost types at the contractual stage as an equilibrium outcome of the game played between the firm and the regulator.

The contribution of the paper is threefold. First we show that cost padding can occur. Secondly we show that investment at the contractual stage is inefficiently low and thirdly we show that if in equilibrium the firm adopts a mixed strategy then the mixed strategy distribution is continuous. Section 3.3 shows that the optimal regulatory contract will tolerate some cost padding. The optimum contract will try to penalize those with high costs and reward those with low costs to reduce cost padding activities. However, since true costs are unknown to the regulator and since reimbursement is socially costly, the regulator will still need to offer higher reimbursement to those with higher costs and won't be able to eliminate cost padding completely. Moreover, we show that because the regulator cannot disentangle true form padded costs, cost padding will introduce a distortion that will restrict the regulated output or raise the regulated price.[8] This result can be related to that small literature which does consider cost-padding in regulated firms. In Albon and Kirby (1983) there is a regulatory constraint such that the firm is not allowed to exceed a given profit target. They assume that padded costs add directly to the utility of the firm and show that it will simply pad costs so that the profit target is met at the monopoly outcome. In Daughety (1984) the regulator imposes a profit target such that profit over revenue cannot exceed some positive fraction less than one. In his model padded costs add nothing to utility but he shows how costs may be padded so that the regulatory constraint is met at the revenue maximizing output where marginal revenue equals zero.[9] In these models however, the regulatory constraint is exogenously given and it is important to reassess this conclusion to see if cost padding might be eliminated when the regulator optimally determines the regulatory environment.[10]

[7]The costly state falsification model has been extended by Maggi and Rodríguez-Clare (1995) who consider a general agency model with risk neutral principal and agent and by Crocker and Morgan (1998) who allow for risk aversion and consider falsification and fraud in insurance contracts.

[8]These results obviously have important legal and policy implications. If evidence of cost padding is found, then this should not be taken as a *prima facia* case of contract violation if the regulatory contract was initially designed to make allowance for some padding of costs.

[9]See Waterson (1988) for a summary of this type of model.

[10]This issue is considered by Laffont and Tirole (1993, chapter 12) in an optimum regulatory framework. However, in their model the firm engages in both cost padding and cost reduction at a post contractual stage and the regulator receives a signal of true costs through auditing.

Section 3.4 shows that when investment in cost reduction is determined at a pre-contractual stage an input distortion is introduced. However, unlike in Averch and Johnson (1962) the effect of the distortion is to reduce investment relative to the cost minimizing level. The reason is that, when there is cost padding, the effect of an increase of investment in reducing true costs is mitigated as lower cost types tend to have higher falsification costs. Thus there is under-investment and higher costs and this exacerbates the problems of cost padding at the contracting stage which are outlined in Section 3.3. In Section 3.4 the properties of the equilibrium type distribution are derived. It is shown that this distribution is continuous. This result is derived rather than imposed as in the model of González (2004). Secondly we show that standard second-order conditions for optimum incentive contracts are satisfied in equilibrium and thus monotonicity restrictions on the hazard rate for types are not required to avoid bunching of types in the optimum regulatory contract. The main advantage of our approach is that cost padding is examined within a standard model of costly state falsification and the investment in cost reduction is tractably addressed using the endogenous screening methodology. It also has the added advantage that the properties of the cost-reimbursement contract depend only on the fundamental technology and preference parameters of the model and do not depend on arbitrary assumptions about the distribution of types which often make such models difficult to test econometrically.

The next section presents the details of the model and Section 3 provides the results. Concluding comments are contained in Section 4

2. The Model

In this section we describe the regulatory model. The regulator wishes to engage a firm to undertake a public project to maximize social welfare. Before the regulator engages the firm, the firm chooses the amount of capital to invest which affects the cost of undertaking the project. Once the firm is engaged it is able to pad its costs. The regulator observes the firm's total cost for the project but does not observe how much capital the firm has invested nor the amount of total costs which are actually padded costs. The regulator has to design a contract for the firm which discourages the firm from padding its costs and encourages the firm to invest in cost reduction. Thus we turn the standard Laffont and Tirole (1993) regulatory model on its head and assume that the effort or investment in cost reduction is pre-contractual and that the firm can artificially inflate or pad its cost at the post-contractual stage after production is undertaken. At the contractual stage the regulator treats the firm's investment in cost reduction as *hidden information* and the padded costs of the firm as a *hidden action*.

The regulator chooses the size of the project q which the firm is to deliver. A project of size q has a social benefit $V(q)$ where $V\colon \mathcal{Q} \to \Re$ where $\mathcal{Q} = [q_{min}, q_{max}] \subset \Re_+$. There is a shadow cost of public funds of $1+\lambda$. We make the standard assumptions that the marginal social benefit is positive but strictly decreasing:

ASSUMPTION 1: $V\colon \mathcal{Q} \to \Re$ is C^3, $V(0)=0$, $V'(q)>0$, $V''(q)<0$ and $\lim_{q\to 0} V'(q) = \infty$. In addition $\lambda > 0$.

If the project is to produce a private good we shall denote the inverse demand function as $P(q)$ where $P\colon \mathcal{Q} \to \Re_+$. In this case $V(q) = (S(q) - R(q)) + (1+\lambda)R(q) = S(q) + \lambda R(q)$ where $R(q) = qP(q)$ is firm revenue and $S(q)$ is consumer surplus. Then the marginal social benefit is $V'(q) = (1+\lambda)P(q) + \lambda q P'(q)$ as $S'(q) = P(q)$.

The firm chooses its capital from an interval $\mathcal{K} = [0, k_{max}] \subset \Re_+$ before the project contract is signed. The market for capital is perfect and the price of capital is $\rho > 0$. The cost to the firm of undertaking the project depends on the size of the project q and the amount of capital k devoted to the project. Denote the firm's variable cost as $g(k,q)$ where $g\colon \mathcal{K}\times\mathcal{Q} \to \Re_+$. The firm's short-run marginal cost is therefore $g_q(k,q)$. We make the following assumptions about the firm's variable cost function:

ASSUMPTION 2: $g\colon \mathcal{K}\times\mathcal{Q} \to \Re_+$ is convex and C^3 with $g_q(k,q) > 0$, $g_k(k,q) < 0$, $g_{qq}(k,q) > 0$, $g_{kq}(k,q) < 0$ and $g_{kk}(k,q) > 0$.

Here we have chosen to take the variable cost function as a primitive of the model. With two minor qualifications we could have taken the firm's production function, say with two inputs, capital k and labor ℓ, as a primitive and derived the properties of the variable cost function given in Assumption 2. The qualifications concern the sign of $g_{kq}(k,q)$ and the continuity of the derivatives of the function at the boundary. Firstly, $g_{kq}(k,q)$ has the same sign as $(f_{\ell\ell} f_k - f_{\ell k} f_\ell)$. In general the later can be of either sign but it is negative for the Cobb-Douglas and CES production functions. Secondly if Inada conditions are imposed on the production function then the derivatives of the cost function, in particular $g_k(k,q)$, may be unbounded and hence not continuous at $k=0$. The later raises a technical issue which we avoid by making assumptions directly on the cost function.[11] We shall also be interested in the special case where $g(k,q)$ can be written as $g(k,q) = g^0(k)g^1(q)$ in which case we shall say that g is *multiplicatively separable* in k and q.

ASSUMPTION 2$'$: There are functions $g^0\colon \mathcal{K} \to \Re_+$ and $g^1\colon \mathcal{Q} \to \Re_+$ such that $g(k,q) = g^0(k)g^1(q)$.

This separability assumption is satisfied if the cost function is derived from the Cobb-Douglas production function. Under Assumption 2$'$ the elasticity of costs with respect to k, i.e. $k\,g_k(k,q)/g(k,q)$, is independent of q.

[11] In the examples we present below the cost functions are derived from a Cobb-Douglas production function. Even though the resultant cost functions have unbounded derivatives, the solutions can be correctly computed.

The regulator cannot observe the true variable cost but instead observes total costs C which also includes the level of cost padding $x \geq 0$ undertaken by the firm, i.e. $C = g(k,q) + x$. Thus the amount x represents the extent to which the firm falsely reports its costs[12] and can be considered as an accounting contrivance that raises the costs as seen by the regulator. We shall also be interested in the proportion of costs padded and let χ satisfy $C = (1+\chi)g(k,q)$ so that χ represents the extent to which input prices are exaggerated by the firm. This is consistent with a number of the examples cited in the introduction where the prices of inputs as billed to the regulator were marked up relative to true cost. We follow the standard costly-state falsification model of Crocker and Morgan (1998) and assume that there is a cost to falsifying the reported costs of $\phi(x)$ that depends on the extent of cost padding x. The following assumption is made about the function $\phi(x)$:

Assumption 3: $\phi\colon \Re_+ \to \Re_+$ is C^3, $\phi(0) = 0$, $0 \leq \phi'(x) \leq 1$, $\phi'(0) = 0$ and $\phi''(x) > 0$.

These assumptions are mainly straightforward. No falsification is costless but even marginal amounts of falsification are costly. The marginal cost of falsification is always positive and increasing so that more exaggerated costs are increasingly costly to falsify. It is assumed that $\phi'(x) < 1$ so that the marginal cost of falsification is always less than the amount falsified. This is made as an assumption here but could easily be imposed as a constraint as it will never be profitable for the firm to falsify the extra unit of cost if the cost of the falsification is greater than any potential benefit. A convenient functional form that satisfies Assumption 3 arises when the elasticity of marginal cost is constant.

Assumption 3$'$: $\phi(x) = \alpha x^{\beta}$ with $\alpha > 0$, $\beta > 1$ provided $x^{\beta-1} < 1/(\alpha\beta)$ for all x.

We now consider the sequential move game in which the firm first chooses its capital input and then the regulator chooses an incentive contract for the firm given the hidden action problem that the firm can pad its costs. We suppose that the firm can chose a mixed strategy and represent its mixed strategy choice by a distribution function $F\colon \mathcal{K} \to [0,1]$. The distribution function is by definition non-decreasing and right-continuous. The distribution may however, have positive density, zero density or positive probability. By the Lebesgue decomposition theorem we know that every distribution function can be written as a convex combination, $F = \alpha_c F_c + \alpha_d F_d + \alpha_s F_s$, where F_c is absolutely continuous, F_d is discrete, F_s is singular continuous (i.e. a distribution which is continuous but non-increasing almost everywhere such as the Cantor distribution) and where $\alpha_d, \alpha_c, \alpha_s \geq 0$ and $\alpha_d + \alpha_c + \alpha_s = 1$. We shall let $F(k-) \equiv \lim_{h \searrow 0} F(k-h)$ be the left hand limit where

[12]There is another interpretation for x as additional or unnecessary expenditures undertaken by the firm and this will be discussed below. As noted in the introduction there are many ways in which firms can pad costs: advertising and sponsorship, transfer of funds across divisions, unnecessary remuneration increases, larger than normal allowances for depreciation, not reporting on cost-saving improvements, and various other perks as well as other costly accounting contrivances.

the notation $\lim_{h \searrow 0}$ indicates that the limit is approached as h tends to zero for $h > 0$. Since the distribution is right continuous $F(k+) \equiv \lim_{h \searrow 0} F(k+h) = F(k)$. Also since the distribution function is monotonic it can have at most a countable set of discontinuities. Let the set of these discontinuities be $\mathcal{M} = \{k_0, k_1, k_2, \ldots\}$. Then we shall let $f(k_i) \equiv F(k_i) - F(k_i-) > 0$ denote the mass or saltus at k_i. If the firm adopts a pure strategy k_0 say, then the distribution has a single mass point with $f(k_0) = 1$. We shall denote the support of F, i.e. all the points where F is strictly increasing, as $\mathcal{S} \subseteq \mathcal{K}$.

Denote $\mathcal{C}$ as the set of possible costs. Given that the regulator can observe the total costs it will offer an incentive contract conditional on $C \in \mathcal{C}$. As a convention suppose that the regulator reimburses the firm's total cost C and that the contract specifies a transfer $t \in \Re$ to the firm as well as the size of the project $q \in \mathcal{Q}$. Note that since reimbursed cost includes padded costs, the transfer t may be (and in some cases will be) negative. The regulator's objective is to maximize total welfare net of costs. Letting π be the profit of the firm, we have that for a given value of profit π and total cost C social welfare is given by

$$V(q) + \pi - (1+\lambda)(t+C).$$

The profit of the firm is given by the transfer t plus the reimbursed costs observed by the regulator C, less the true variable costs of production $g(k,q)$, less the cost of falsifying the accounts $\phi(C - g(k,q))$, less capital costs. Thus

$$\pi = t + C - g(k,q) - \phi(C - g(k,q)) - \rho k.$$

For convenience we denote the firm's rent from the contract as $r = \pi + \rho k$ and the costs incurred by the firm as $v(C,k,q) = g(k,q) + \phi(C - g(k,q))$ where $v\colon \mathcal{C} \times \mathcal{K} \times \mathcal{Q} \to \Re_+$. Given Assumption 2 and Assumption 3, the function v is strictly decreasing and strictly convex in k and satisfies a Spence-Mirrlees *single crossing property*, which for convenience we state as a separate assumption.

Assumption 4 (Single Crossing Property): The marginal cost v_k satisfies the single crossing property with $v_{kC} > 0$ and $v_{kq} < 0$.

With these definitions for the rent of the firm r, and the firm's cost v, social welfare is

$$V(q) + \pi - (1+\lambda)(t+C) = V(q) - (1+\lambda)v(C,k,q) - \lambda r - \rho k.$$

The first term on the right hand side of the equality is the social benefit of the project and the second term is total social cost. The third term is the social cost of giving the firm a rent and the final term is the capital cost. We shall let $\omega(C,k,q) = V(q) - (1+\lambda)v(C,k,q)$ denote the net social benefit of the project. To ensure that the regulator's optimization problem is concave we shall make a further assumption.

Assumption 5: Both the benefit function $\omega(C,k,q) = V(q) - (1+\lambda)v(C,k,q)$ and the marginal cost function $v_k(C,k,q)$ are assumed to be strictly concave in C and q.

Assumption 5 clearly implies further restrictions on the benefit and cost functions beyond those of Assumption 1, 2 and 3 and imposes restrictions on the third derivatives of the cost function g and cost falsification function ϕ (all of which are satisfied by the examples we compute below). In particular Assumption 5 implies that $\phi''' \leq 0$ so that the marginal falsification costs are also concave. Moreover Assumption 5 is sufficient to assure that random contracts are sub-optimal.

As we have stated although the regulator observes C this does not allow any inferences to be made about the level of capital k chosen by the firm as the cost padding x is also unobservable. We therefore assume that the regulator forms a probability assessment about the choice of capital input made by the firm and represent this by a probability distribution $F^e\colon \mathcal{K} \to [0,1]$ with support $\mathcal{S}^e$. Given this probability assessment the regulator will design the transfer and project size to maximize expected social welfare by making the transfer t and size q depend upon the observed costs C so as to reduce the inefficiencies caused by the firm's potential to cost pad. This is a very standard hidden action problem and it is possible to apply the revelation principle to restrict attention to direct mechanisms where the firm reports its investment in cost reduction k' and impose incentive compatibility constraints that the firm has no incentive to misreport its investment level. We shall say that the regulator's probability assessment is *consistent* if $F^e = F$.[13] Therefore the regulator's problem can be formulated by allowing the regulator to choose a contract $\Delta \equiv (C(k), q(k), r(k))_{k\in\mathcal{K}}$ which specifies the cost $C(k)$, the project size $q(k)$ and the rent $r(k)$ as functions of investment k to maximize expected social welfare

$$\int_{\mathcal{K}} \{V(q) - (1+\lambda)v(C,k,q) - \lambda r(k) - \rho k\}\, dF^e(k) \tag{1}$$

subject to incentive compatibility and participation constraints for the firm. To specify the incentive and participation constraints, define $r(k',k) = t(k') + C(k') - v(C(k'),k,q(k'))$ to be the rent which the firm earns from the contract when the actual investment is k and when the firm reports that it has invested k'. Let $r(k) = r(k,k)$. The firm will choose to announce the level of cost reducing investment k' that maximizes the rent given the contract Δ it faces. Therefore the incentive compatibility constraints are

$$r(k) \geq r(k',k) = t(k') + C(k') - v(C(k'),k,q(k')) \qquad \forall\ k,k' \in \mathcal{S}^e. \tag{2}$$

Given incentive compatibility and since the investment in cost reduction is a sunk cost, for the firm to participate in the contract it must always derive a non-negative rent for any

[13]For an analysis of screening where the beliefs of the principal and agent are not consistent see Grubb (2005).

investment level k actually made. That is

$$r(k) \geq 0 \qquad \forall\ k \in \mathcal{S}^e. \tag{3}$$

An equilibrium will involve a regulatory contract which maximizes social welfare subject to the incentive compatibility and participation constraints, a strategy for the firm which maximizes profits and a consistent probability assessment. If the firm is to adopt a mixed strategy in equilibrium, each possible choice of investment level k must also give rise to the same level of profit. That is

$$r(k) - \rho k = r(k') - \rho k' \qquad \forall k, k' \in \mathcal{S}. \tag{4}$$

As is standard we shall further assume that if the firm is indifferent between two or more strategies then it will choose the strategy that maximizes the regulator's objective. Hence we can define an equilibrium as follows:

Definition 1: A equilibrium will be a contract Δ which maximizes (1) subject to (2) and (3), a strategy for the firm that maximizes profits and a consistent probability assessment $F^e = F$. Where the firm is indifferent between two or more strategies it chooses the strategy which maximizes the regulator's payoff. In addition if the firm chooses a mixed strategy equation (4) is also satisfied.

Before proceeding to the analysis it is important to realize that although in describing the model we have treated cost padding as an accounting contrivance, there is an equivalent model in which cost padding can be interpreted as an extra but unnecessary expenditure that generates some utility benefit for the firm. To see this let $\psi(x)$ denote the benefit to the firm of cost padding an amount x. In this case ψ may be thought of as the benefit of gold-plating expenditures. Mirroring the assumptions on the cost of falsification $\phi(x)$ assume that the benefit function $\psi(x)$ satisfies $\psi(0) = 0$, $0 \leq \psi'(x) \leq 1$, $\psi'(0) = 1$ and $\psi''(x) < 0$. That is an increase in padded costs by one unit generates a positive gain in utility (but not as much as the costs incurred) and marginal benefit is declining with costs. The rent of the firm r, is simply given by the transfer t plus the utility benefit of the padded costs $\psi(x)$ so that$r = t + \psi(x)$. Then the two alternative interpretations are formally equivalent if $\psi(x) = x - \phi(x)$ as in this case

$$r = t + C - g(k,q) - \phi(C - g(k,q)) = t + x - \phi(x) = t + \psi(x).$$

Although the two interpretations of cost padding are formally equivalent when $\psi(x) = x - \phi(x)$ it should be emphasized that these two alternatives represent very different situations. In one case there is a real expenditure that generates utility benefits whereas in the other case it is an accounting contrivance which has real costs.

3. Results

The results are organized as follows. Section 3.1 considers the benchmark first-best solution where the regulator can observe the true cost. Section 3.2 develops some preliminary results so that the optimum contract can be examined. Section 3.3 assumes that the firm adopts some mixed strategy and derives the optimum contract given that mixed strategy and Section 3.4 extends the analysis by deriving the equilibrium distribution. Finally Section 3.5 examines whether a pure or mixed strategy for the firm maximizes welfare. All omitted proofs are given in an Appendix.

3.1. The First-Best

As a benchmark consider the case where the regulator can observe the true cost (and hence can infer k) so there is no hidden action or hidden information problem. In this case there is no cost padding and the first-best optimum is to set the marginal social value of an extra unit of output equal to the short run marginal cost of public funds. The short run marginal cost of public funds is $(1+\lambda)g_q(k,q)$ and the condition for the first-best optimum is

$$V'(q) = (1+\lambda)g_q(k,q). \tag{5}$$

In addition the firm will choose the capital input to equate the marginal benefit of cost reduction for a given project level to its price

$$-g_k(k,q) = \rho. \tag{6}$$

We shall let k^* and q^* denote the first-best levels of capital and project size that satisfy (5) and (6). It will also be useful to denote the first-best level of output for a given capital input which solves (5) as $q^{FB}(k)$. Likewise denote the first-best level of investment for any given output level which satisfies (6) as $k^{FB}(q)$. It is straightforward to see from Assumption 1 and Assumption 2 that both $q^{FB}(k)$ and $k^{FB}(q)$ are strictly increasing. Also by definition $q^* = q^{FB}(k^*)$ and $k^* = k^{FB}(q^*)$.

Lemma 1: *Given Assumption 1 and Assumption 2, there is a unique pair* (k^*, q^*) *with* $q^* > 0$ *which solves equations (5) and (6).*

Proof: Solving for $k^{FB}(q)$ from (6) and substituting into (5) it follows from the strict concavity of V and the convexity of g that the $V'(q) - (1+\lambda)g_q(k^{FB}(q), q)$ is strictly decreasing in q. Since $\lim_{q\to 0} V'(q) = \infty$, $q^* > 0$. □

For a private good, let the Lerner index be given by $L = (p - g_q(k,q))/p$ and the Ramsay index be given by $R = (\lambda/(1+\lambda))\eta^{-1}$ where $\eta = -\frac{dq}{dp}/\frac{q}{p}$ is the elasticity of demand. At the social optimum (k^*, q^*), the Ramsay pricing rule $L = R$ holds and the

Lerner index is proportional to the inverse of the elasticity of demand with the factor of proportionality $\lambda/(1+\lambda)$.

3.2. Preliminaries

First consider an equilibrium in which the firm chooses a pure strategy. If the firm were to choose a pure strategy, then the regulator's best response would be to offer a contract that simply reimbursed the known costs of the firm's investment choice, i.e. to offer a fixed price contract and choose the project size at the first-best level of $q^{FB}(k)$. In such a situation no information rent is paid to the firm and no costs are padded, $r(k) = 0$ and $C(k) = g(k, q^{FB}(k))$. In these circumstances the best strategy for the firm to maximize profits is to choose $k = 0$, so that the firm has zero profits and social surplus is $V(q^{FB}(0)) - (1+\lambda)g(0, q^{FB}(0))$.

We shall now consider situations in which the firm plays a mixed strategy F and shall later compare the equilibria with pure and mixed strategies. For such a mixed strategy it is easy to establish the standard result that in any optimal contract the local downward incentive compatibility constraints from equation (2) are binding.

LEMMA 2: *Given a distribution for the regulator's beliefs F^e then for any two isolated but adjacent values in the support, the downward incentive compatibility constraints bind at the optimum contract.*

Using this property it then follows that if the firm adopts a mixed strategy, the support of the regulator's beliefs must be an interval in equilibrium and hence a connected set. Thus we can conclude that $\alpha_s < 1$ so the distribution cannot just be composed of a singular continuous element (which has a nowhere dense support). The idea is simple. If the support is not an interval then the firm can choose an intermediate investment level not in the support, report a lower investment level within the support and increase profits.

LEMMA 3: *If the regulator's beliefs are consistent then the set $\mathcal{S}^e$ is a closed interval.*

Having shown that the support $\mathcal{S}^e$ is a closed interval denote its lower endpoint by $\underline{k}$ and its upper endpoint by $\overline{k}$. We shall determine the values of $\underline{k}$ and $\overline{k}$ in Section 3.4. Since the continuous part of the distribution F_c is monotonic on this interval it is differentiable almost everywhere. As normal we can define a density function f_c for the continuous part of the distribution function to equal the derivative F_c' where the derivative exists and zero otherwise. Then given the support is an interval, we can write the regulator's objective function as

$$(1') \qquad \int_{\underline{k}}^{\overline{k}} \left(V(q(k)) - \lambda r(k) - (1+\lambda)v(C(k), k, q(k)) - \rho k\right) dF^e(k)$$

where the differential is

$$dF^e(k) = \begin{cases} F^e(k_i) - F^e(k_i-) & \text{if } k_i \in \mathcal{M}^e \\ f_c^e(k) & \text{if } k \in \mathcal{S}^e/\mathcal{M}^e \end{cases}$$

Further since the support is an interval and hence connected we can use the fairly standard procedure (see e.g. Segal and Whinston (2002)) to give necessary and sufficient conditions for incentive compatibility.

LEMMA 4: *Necessary and sufficient conditions for incentive compatibility are*

(i) $r(k) = r(\underline{k}) - \int_{\underline{k}}^{k} v_k(C(\kappa), \kappa, q(\kappa))\, d\kappa$

(ii) $-\int_{k}^{k'} (v_k(C(k'), \kappa, q(k')) - v_k(C(\kappa), \kappa, q(\kappa)))\, d\kappa \geq 0 \quad \forall\, k \text{ and } k'.$

The first part of Lemma 4 shows that higher rent must be paid to a low cost firm (high k) to induce them to report their lower costs since $v_k(C,k,q) < 0$. This higher rent will exactly reflect the reduction in costs. As k is increased by one unit true cost falls by $g_k(k,q)$, but for a given C there is an increase in falsification costs of $\phi'(C - g(k,q))g_k(k,q)$, thus the rent should rise by $v_k(C,k,q(k)) = g_k(k,q)(1 - \phi'(C - g(k,q)))$. Moreover Lemma 4(ii) shows that the derivative $\dot{r}(k)$ satisfies

$$\dot{r}(k) = -v_k(C(k), k, q(k)) \tag{2$'$}$$

wherever $r(k)$ is differentiable. The second part of Lemma 4 is the usual second-order condition expressed in integral form. We shall follow the standard procedure of ignoring this condition and then checking it is satisfied by the optimum contract. Thus equation (2′) replaces all the incentive compatibility conditions of equation (2). Likewise it is possible to follow a standard argument to simplify the participation constraint given in equation (3). Since the rent enters negatively into the objective function (1) with $\lambda > 0$ and since $\dot{r}(k) \geq 0$ from (2′), the regulator gains by a parallel downward shift in the rent function $r(k)$ so that the participation constraints can be replaced by the single equality condition

$$r(\underline{k}) = 0. \tag{3$'$}$$

Taking these simplifications into account the regulator's problem is to choose a contract $\Delta = (C(k), q(k), r(k))$ to maximize (1′) subject to (2′) and (3′). Integrating $r(k)dF^e(k)$ by parts, using Lemma 4(ii), (2′) and (3′) and substituting into (1′) gives the virtual surplus function

$$\int_{\underline{k}}^{\overline{k}} (V(q) - (1+\lambda)v(C,k,q) - \rho k)\, dF^e(k) + \int_{\underline{k}}^{\overline{k}} \lambda v_k(C,k,q)(1 - F^e(k))dk. \tag{1$''$}$$

3.3. *Optimal Contract with Fixed Distribution*

Since we shall only consider consistent distributions we henceforth drop the superscript notation e to distinguish the regulator's beliefs from the actual distribution of k. The following first-order conditions are derived straightforwardly from equation ($1''$)

$$(1+\lambda)v_C(C,k,q)dF(k) = \lambda v_{kC}(C,k,q)(1-F(k)) \tag{7}$$

$$\left(V'(q)-(1+\lambda)v_q(C,k,q)\right)dF(k) = -\lambda v_{kq}(C,k,q)(1-F(k)). \tag{8}$$

These two equations can be written more illuminatingly as

$$(1+\lambda)\phi'(C-g(k,q))dF(k) = -\lambda(1-F(k))g_k(k,q)\phi''(C-g(k,q)) \tag{7$'$}$$

$$\left(V'(q)-(1+\lambda)g_q(k,q)\right)dF(k) = -\lambda(1-F(k))(1-\phi'(C-g(k,q)))g_{kq}(k,q) \tag{8$'$}$$

where equation ($8'$) has been simplified by using both equations (7) and (8). At the first-best solution the left-hand-sides of both equations ($7'$) and ($8'$) are equal to zero. Thus the equations show how the optimal contract differs from the first-best solution. Consider first equation ($7'$). The benefit of reducing cost padding by one unit is simply the saving in falsification costs $\phi'(x)$. If cost padding is reduced for $dF(k)$ firms, the benefit to society is $(1+\lambda)\phi'(x)dF(k)dk$ as the shadow cost of the funds saved is $(1+\lambda)$. The cost of this one unit reduction is however, the extra rent determined by equation (2) that must be paid to all firms in the interval $[k,\bar{k}]$ to induce them to report lower costs. The change in the rent for a firm with capital k following a one unit reduction in padded costs is $\phi''(x(k))g_k(k,q(k))dk$. The social cost of an extra rent payment of one unit is λ and since $(1-F(k))$ firms are affected the social cost of the extra rent payments is $\lambda(1-F(k))\phi''(x(k))g_k(k,q(k))$. Equating the marginal social cost to the marginal social benefit gives equation ($7'$).

The interpretation of equation ($8'$) is equally straightforward. A small change in q changes the social benefit by $V'(q)$ and changes social cost by $(1+\lambda)g_q(k,q)$ for any given level of capital, k. The extra term in equation ($8'$) represents the marginal increase in rent needed by the more efficient firms to maintain incentive compatibility. The effect on the rent of more efficient firms is $(\phi'(x)-1)g_{kq}(k,q)$ which is positive for a small increase in output given Assumption 2 that more efficient firms have lower marginal costs as well as lower average costs, i.e./ $g_{kq}(k,q) < 0$. This cost of extra rent has to be weighted by the social cost λ and the proportion of more efficient firms there are, $1-F(k)$, relative to the proportion of firms, $dF(k)$, with capital k. Thus equating the marginal social benefit to the marginal social cost plus the marginal social incentive cost gives equation ($8'$). We can therefore conclude from these two equations the following proposition.

PROPOSITION 1: *Given Assumptions 1, 2 and 3, if there is an equilibrium in which the firm adopts a mixed strategy $F(k)$, then costs are padded, $x(k) = C(k) - g(k, q(k)) > 0$, and output is restricted, $q(k) < q^{FB}(k)$, for each $k < \bar{k}$.*

PROOF: Follows since $dF(k) > 0$ and $F(k) < 1$ for $k < \bar{k}$. Since $\phi''(x) > 0$ and $g_k(k, q(k)) < 0$, then $\phi'(x) > 0$ and costs are padded. To see that output is restricted recall that $\phi'(C(k) - g(k, q(k))) < 1$ and we have assumed that $g_{qk}(k, q(k)) < 0$ so that the marginal social incentive cost is positive. Thus

$$V'(q(k)) - (1 + \lambda)g_q(k, q(k)) > V'(q^{FB}(k)) - (1 + \lambda)g_q(k, q^{FB}(k))$$

for $k < \bar{k}$. Then since $V''(q) < 0$ and $g_{qq}(k, q) > 0$, we have that output is restricted below the efficient level. □

It is important to remember that although Proposition 1 shows that costs are padded and output restricted below the first-best this does not necessarily mean that the firm does not cost minimize and there need be no distortion of input choices (the next section will show how input choices may be distorted). It should also be noticed that the regulator will take into account that costs will be padded when setting the contract. Thus a firm that reports a high cost (low k) may be required to make a transfer to the regular to offset the fact that the regulator reimburses full costs. On the other hand a firm that reports a low cost (high k) may well receive a positive transfer from the regulator.

Equation (7′) also provides information on when there will be no cost padding. Clearly there is no cost padding if $g_k(k, q) = 0$ or $\phi''(x) = 0$. The former follows directly as if the unobserved investment does not influence costs, then the regulator will know the true cost and there is no room for cost padding. However, there will also be no cost padding if the costs of falsification are linear. This result has been demonstrated by Lacker and Weinberg (1989) who assume a cost function of the type $\phi(x) = \alpha\,|x|$. The intuition is that if falsification costs are linear then the cost of deterring cost padding is the same independently of the amount of cost padding and therefore no extra rent has to be offered to firms higher in the distribution. Thus if any cost padding is to be deterred this will also deter very small amounts of cost padding as well. Also since $\bar{k}$ is in the support, $dF(\bar{k}) > 0$ so there is also the classical efficiency at the top result that there is no cost padding for $k = \bar{k}$. Hence output is also at the first-best $q(\bar{k}) = q^{FB}(\bar{k})$ for $k = \bar{k}$. The intuition is that there is always some benefit to reducing falsification at the very highest capital level but the social cost is infinitesimal as no extra rent has to be paid to any other firm type.

For a purely private good the restriction in output can be expressed in terms of the Lerner and Ramsay indices by rewriting equation (8′) as

$$L = R - \frac{\lambda}{(1+\lambda)}\frac{1}{\eta}\left(\frac{(\phi'(C(k)-g(k,q(k)))-1)g_{qk}(k,q(k))(1-F(k))}{pdF(k)}\right).$$

so the effect of cost padding is to raise the Lerner index above the Ramsay index. Thus cost padding under optimal regulation will tend to restrict output and raise the price. The intuition is fairly simple, since the regulator faces the problem of the firm falsifying its cost upward, it can keep total costs lower by restricting output below the normal Lerner-Ramsay rule, that is by relaxing the price regulation on the firm. This is a similar conclusion to that found in Daughety (1984) with an arbitrary regulatory constraint and applies in an optimum regulator environment under standard assumptions on firm technology.

Under Assumptions 2′ and 3′ equations (7′) and (8′) can be treated independently and hence we have the result that the size of the project does not effect the extent to which input costs are exaggerated.

Proposition 2: *Under Assumptions 2′ and 3′, if there is an equilibrium in which the firm adopts a mixed strategy $F(k)$, then the proportion of costs padded χ is independent of q.*

Proof: Using Assumptions 2′ and 3′ in equation (7′), and recalling that $C = (1+\chi)g(k,q)$ it can be seen that χ is independent of q. □

We conclude this sub-section with a short example to illustrate the nature of the optimum contract.

Example 1: Short-run variable costs are $g(k,q) = \frac{q^2}{4k}$. Falsification costs are quadratic, $\phi(x) = \frac{1}{2}x^2$ and that the distribution function is uniform on $[0,1]$. Demand is unit elastic, $P(q) = \frac{1}{q}$ and the shadow cost of public funds is $\lambda = 1$. Then the marginal social benefit of the project is $\frac{1}{q}$ and the marginal social cost is $\frac{q}{k}$ which declines with k. Equation (7′) shows that $\chi(k) = \frac{1}{2}\frac{1-k}{k}$. Hence low cost firms will engage in proportionally less cost padding. The marginal social incentive costs are defined for $q < \frac{2\sqrt{2k}}{\sqrt{1-k}}$ and are given by

$$\frac{(1-k)q}{2k^2}\left(\frac{1}{2}\frac{(1-k)q^2}{4k^2}-1\right).$$

Then equating marginal social benefit with the marginal social cost plus the marginal social incentive cost gives $q(k) = \frac{2k}{1+\sqrt{k}}$.[14] Total costs are $C(k) = \frac{1+k}{2(1+\sqrt{k})^2}$ and the transfer

[14]It can be shown from the solutions for $\chi(k)$ and $q(k)$ that the second-order condition given in Lemma 4(ii) are satisfied.

function is given by

$$t(k) = -\frac{3 + 26\sqrt{k} + 27k - 24(1+\sqrt{k})^2 \log_e(1+\sqrt{k})}{8(1+\sqrt{k})^2}.$$

A numerical calculation shows that transfers are negative for $k < 0.317818$ and positive thereafter.

3.4. *Equilibrium Distribution*

As discussed in Section 2 if the firm is to adopt a mixed strategy the profit of the firm must be the same for each level of investment in the support. That is the rent of the firm satisfies $r(k) = const. + \rho k$ for every k in the support. Since by Lemma 3 the support is an interval and as the profits of the firm must be the same for each possible level of cost reduction we can use the incentive compatibility condition of equation (2) to derive

$$-v_k(C(k), k, q(k)) = -(1 - \phi'(C(k) - g(k, q(k))))g_k(k, q(k)) = \rho. \tag{9}$$

This equation has some straightforward but important implications.

PROPOSITION 3: *If there is an equilibrium where the firm adopts a mixed strategy, then investment is below the first best, $k < k^{FB}(q)$ for all $k < \bar{k}$. Moreover if costs are derived from cost minimization subject to a production constraint, the marginal rate of technical substitution of capital for labor exceeds its relative price.*

PROOF: First for a given level of output q, equation (9) shows that $k < k^{FB}(q)$ for all $k < \bar{k}$, since $\phi'(C(k) - g(k, q(k))) < 1$ for $k < \bar{k}$ from equation (7′) and $g_{kk}(k, q) > 0$. Equally if cost is derived from a production function, say with other input of labor ℓ then $-g_k(k, q) = \frac{f_k}{f_\ell}$ and since equation (9) shows that $-g_k(k, q) > \rho$ costs are higher than the first-best level for any given output and investment. □

Thus investment is lower than the cost minimizing level for all output levels except $q(\bar{k})$. This is a reverse Averch-Johnson effect that depresses capital accumulation relative to its cost minimizing level. This is in contrast to the model of Averch and Johnson (1962) where the marginal rate of technical substitution of capital for labor is less than the relative price of capital, $\frac{f_k}{f_l} < \rho$ as the firm over accumulates capital to meet a rate of return constraint on capital.

Next we consider the determination of the endpoints of the support of the distribution, $\underline{k}$ and $\bar{k}$.

LEMMA 5: *In any equilibrium in which the firm adopts a mixed strategy $\underline{k} = 0$ and $\bar{k} = k^*$.*

PROOF: At the pre-contractual stage the firm has the option not to undertake any extra investment in cost reducing activity. With k known the regulator will leave the firm with no profits. The firm can do no worse than this, so there is an ex-ante constraint that $r(k) - \rho k \geq 0$. Since $r(\underline{k}) = 0$ from equation ($3'$), this implies $\underline{k} = 0$, so that the lower end of the distribution represents no *ex ante* investment in cost reducing activity. Clearly if the firm were to choose a $k > k^*$ with positive probability, there would be an advantage to reduce $\overline{k}$ by dk leading to a cost saving of more than ρdk. The only efficiency benefit from not doing so would be if there were additional efficiency savings from a reduction in falsification costs, but since as we have already argued there is no cost padding at $\overline{k}$, there are no such benefits and it is better to reduce $\overline{k}$ to k^*. □

That $\underline{k} = 0$ is a manifestation of the hold-up problem for the firm's investment in cost reduction. Once the firm has invested in cost reducing activity at the pre-contractual stage, the regulator can extract the entire rent from the highest cost firm and thus in order to have non-negative profits ex ante, no extra investment in cost reduction must be an option for the firm. Thus we can conclude from the participation constraint of equation ($3'$) that the rent function satisfies $r(k) = \rho k$.[15] The upper endpoint of the distribution can never be inefficiently high because of the efficiency at the top property and hence at the upper endpoint $\overline{k} = k^*$ and $q = q^*$.

We can now show that the equilibrium distribution has no mass points. Using equation (9) and integrating by parts the regulator's virtual surplus given by equation ($1''$) can be rewritten as

$$\int_0^{k^*} \omega(C, k, q)\, dF(k) - (1 + \lambda)\rho \int_0^{k^*} (1 - F(k))\, dk. \tag{10}$$

The virtual surplus then consists of two terms. The first term is the expected net social benefit. The second term is the social capital costs which reflect the extra rent that has to be paid to firms with higher investment. These costs depend on the probability $(1 - F(k))$ of having investment higher than k and the social cost of capital $(1 + \lambda)\rho$. With this observation it is possible to prove the following.

PROPOSITION 4: *In any equilibrium in which the firm adopts a mixed strategy there are no mass points*

Proposition 4 shows that $\alpha_d = 0$ so that the equilibrium distribution is continuous. The idea of the proof is to consider a distribution with mass points and to show that a change in the distribution that smooths out the discontinuity by appropriately shifting the distribution function but keeping the original contract can increase the virtual surplus.

[15] Since the rent just covers the investment cost it may be viewed as a rent to the quasi-fixed factor at the contractual stage rather than an information rent.

Clearly then changing the contract to the optimum for the new distribution will increase the virtual surplus further or at least not decrease it. It is clear from equation (10) that a change to a stochastically dominant distribution increases expected social capital costs but so too will expected net social benefit if $\omega(C(k), k, q(k))$ is increasing in k. The proof of Proposition 4 works by showing that on any continuous part of the distribution the net social benefit function ω increases in k at a rate greater than $(1+\lambda)\rho$, so that an appropriate change in the distribution increases net social benefit by more than the social capital costs and hence will increase the virtual surplus.

The optimum contract Δ and equilibrium distribution $F(k)$ will be determined by equations (9), (2′), (7′) and (8′) together with equation (2′) and the endpoint conditions for the distribution. This system of equations is in general quite difficult to analyze so to illustrate how the distribution and contract are simultaneously determined in equilibrium consider a procurement variant of the model where output is fixed. Normalizing the fixed output level to unity, equation (9) is rewritten as $-(1-\phi'(C-g(k,1)))g_k(k,1) = \rho$ and can be used to determine $C(k)$ directly. From the implicit function theorem $C(k)$ is both continuous and differentiable and it can be seen that $C(k)$ is decreasing and hence that the second-order condition for the incentive constraint is satisfied. The hazard rate is given by the function $h(k) = F'(k)/(1-F(k))$. The hazard rate is then determined by equation (7′) alone, which using the above equation gives

$$h(k) = -\frac{\lambda}{1+\lambda}\frac{\phi''(C(k)-g(k,1))g_k(k,1)}{\phi'(C(k)-g(k,1))}. \tag{11}$$

It should also be noticed that since equation (9) implies that the condition of Lemma 4(ii) is satisfied, there is no bunching in equilibrium and there is no need to impose a monotonicity assumption on the hazard rate to guarantee this result as is normally needed when the distribution of types is taken as fixed. The next example shows how to compute such a solution.

Example 2: $g(k,1) = 2(1-\sqrt{k})$; $\phi(x) = \frac{1}{2}x^2$, $\lambda = \rho = 1$. Then $k^* = 1$ so the distribution function is defined on $[0,1]$. Next equation (9) shows that $x(k) = 1-\sqrt{k}$ and $C(k) = 3(1-\sqrt{k})$. Thus from equation (11) the hazard rate is $h(k) = \left(2(1-\sqrt{k})\sqrt{k}\right)^{-1}$ and the distribution function is $F(k) = \sqrt{k}$.[16] The transfer function is $t(k) = \frac{1}{2}(3k-1)$ and the transfer as a function of observed costs is $t(C) = 1 - C + \frac{1}{6}C^2$.

[16]If this distribution function were used in the case where k is fixed then the resulting contract would involve bunching of types as the corresponding hazard rate is non-monotonic. Since k is chosen endogenously there is no bunching of types.

3.5. *Optimality of Mixed Strategy*

Section 3.3 has determined the optimal contract assuming that the firm adopts some given mixed strategy and Section 3.4 has shown how to find the equilibrium mixed strategy. However this solution needs to be compared with the solution if the firm adopts a pure strategy. We have shown in Section 3.2 that if the firm adopts a pure strategy it will choose $k = 0$ and the social welfare generated will be $V(q^{FB}(0)) - (1+\lambda)g(0, q^{FB}(0))$. Since both the pure strategy and mixed strategy equilibrium deliver the same level of expected profit for the firm, the two should be compared by the social welfare they generate. The level of welfare in the mixed strategy case has been derived in Section 3.4 and is given by equation (10). This comparison is non-trivial since in the pure strategy case there is no cost padding and hence $q^{FB}(0) > q(0)$ whereas the mixed strategy equilibrium will also involve some probability of higher investment and some probability of the efficient or close to the efficient investment level and thus lower costs. We shall simply show by way of an example that the mixed strategy equilibrium can dominate in terms of welfare and since we have assumed that the firm will choose the outcome preferred by the regulator if it is indifferent, the mixed strategy will then be the equilibrium outcome.

Example $2'$: This example is the same as Example 2. Let the social benefit of the project be $V > 3$. The welfare generated in the pure strategy case is $V - (1+\lambda)g(0,1) = V - 2(1+\lambda)$. In the mixed strategy case with the equilibrium distribution $F(k) = \sqrt{k}$ on $[0,1]$ and cost padding $x(k) = 1 - \sqrt{k}$, equation (10) gives the welfare as

$$V - (1+\lambda)\int_0^1 \left(2(1-\sqrt{k}) + \frac{1}{2}(1-\sqrt{k})^2\right)\frac{1}{2\sqrt{k}}\,dk - (1+\lambda)\rho\int_0^1 (1-\sqrt{k})\,dk.$$

With $\lambda = 1$ and $\rho = 1$ welfare is $V - 2(\frac{7}{6}) - 2(\frac{1}{3}) = V - 3 > V - 4$. So the mixed strategy generates greater welfare. Notice that if $V \in (3,4)$ then only with the the mixed strategy is the project worth undertaking.

4. Conclusion

This paper has extended the standard model of regulated monopolies to allow for both cost padding and incentives for cost reduction. It has been assumed that the cost reducing activity is undertaken at a pre-contractual stage and any cost padding is undertaken post-contractually. It is shown how some cost padding will be tolerated in optimal regulatory contracts. It also induces a move away from Ramsay pricing and implies weaker price regulation than without cost padding in the case where the project produces a private good. By allowing firms to undertake a pre-contractual investment in cost reduction, the distribution of cost types is derived endogenously and therefore the properties of the cost-reimbursement contract depend only on the fundamental technology and preference parameters of the model and do not depend on arbitrary assumptions about the distribution of types which often make such models difficult to test econometrically. It has been shown that although costs are padded short-run input choices need not be distorted from their cost minimizing levels whereas the pre-contractual investment in cost reduction is distorted below its cost minimizing level.

Appendix

Lemma 2: *Given a distribution for the regulator's beliefs F^e then for any two isolated but adjacent values in the support, the downward incentive compatibility constraints bind at the optimum contract.*

Proof: Consider two adjacent values k^+ and k^- where $k^+ > k^-$. Suppose that $r(k^+, k^+) > r(k^-, k^+)$ contrary to the assertion in the lemma. That is suppose

$$t(k^+) + C(k^+) - v(C(k^+), k^+, q(k^+)) > t(k^-) + C(k^-) - v(C(k^-), k^+, q(k^-)).$$

From the incentive compatibility conditions of equation (2) it is easy to check that $r(\hat{k}) > r(k^+)$ for all $\hat{k} > k^+$ as $v(C, k, q)$ is decreasing in k. Also $r(\tilde{k}, k^+) > r(\tilde{k}, \hat{k})$ again since $v(C, k, q)$ is decreasing in k. This implies that the local downward incentive compatibility constraints imply the constraints for all higher values of k are also satisfied. Now suppose the regulator lowers $t(k^+)$ so that $r(k^+, k^+) = r(k^-, k^+)$ and lowers $t(\hat{k})$ by the same amount for all $\hat{k} > k^+$. This change will not affect incentive compatibility since we know that the constraints for higher k remain satisfied and it otherwise either relaxes the constraints or leaves them unchanged. Such a change increases the regulator's welfare and so in any optimum the adjacent downward incentive constraints bind. □

Lemma 3: *If the regulator's beliefs are consistent then the set $\mathcal{S}^e$ is an interval.*

Proof: The support of a distribution is the smallest closed set whose complement has probability zero. So suppose to the contrary that $\mathcal{S}^e$ is not an interval. Then there are values k, k' and k'' with $k' < k < k''$ such that $k', k'' \in \mathcal{S}^e$ and $k \notin \mathcal{S}^e$. Let $k^- = \sup\{k' \in \mathcal{S}^e : k' < k\}$ and $k^+ = \inf\{k'' \in \mathcal{S}^e : k'' > k\}$. From Lemma 2 in an optimal contract the local downward incentive compatibility constraint between k^+ and k^- binds. We shall now show that given such an optimum contract, the firm can always deviate and raise profits by choosing some k between any k^- and k^+ and reporting k^-. Let π^- denote profit, t^- denote the transfer and so on when k^- is chosen. Then

$$\begin{aligned} \pi^- &= t^- + C^- - v(C^-, k^-, q^-) - \rho k^- \\ \pi^+ &= t^- + C^- - v(C^-, k^+, q^-) - \rho k^+ \end{aligned}$$

where the second equation uses the fact that the downward incentive compatibility constraint binds. Now suppose that the firm chooses some $k \in (k^-, k^+)$. Let $\pi(k^-, k)$ denote the level of profits from choosing k but reporting k^-. For this still to be an equilibrium the original contract must remain incentive compatible, $\pi(k^-, k) < \pi^- = \pi^+$. Using the previous equations

$$\begin{aligned} \pi(k^-, k) - \pi^- &= v(C^-, k^-, q^-) - v(C^-, k, q^-) - \rho(k - k^-) \\ \pi(k^-, k) - \pi^+ &= v(C^-, k^+, q^-) - v(C^-, k, q^-) + \rho(k^+ - k). \end{aligned}$$

Therefore if the firm is not to gain from deviation the following conditions must hold

$$\begin{aligned} v(C^-,k^-,q^-) - v(C^-,k,q^-) &< \rho(k-k^-) \\ v(C^-,k,q^-) - v(C^-,k^+,q^-) &> \rho(k^+-k). \end{aligned}$$

Taking any k such that $k^- < k \leq \frac{1}{2}(k^- + k^+)$ and given that v is strictly decreasing this implies that

$$0 < v(C^-,k^-,q^-) - v(C^-,k,q^-) < \rho(k-k^-) \leq \rho(k^+-k) < v(C^-,k,q^-) - v(C^-,k^+,q^-).$$

But for $k = \frac{1}{2}(k^- + k^+)$ this contradicts the strict convexity of v in k which follows from Assumptions 2 and 3. Thus if $\mathcal{S}^e$ is not an interval the firm will always gain by deviating and choosing some investment level not in $\mathcal{S}^e$ between the two adjacent points in $\mathcal{S}^e$ and reporting the lower investment level (higher cost). □

LEMMA 4: *Necessary and sufficient conditions for incentive compatibility are*

(i) $r(k) = r(\underline{k}) - \int_{\underline{k}}^{k} v_k(C(\kappa),\kappa,q(\kappa))\,d\kappa$

(ii) $-\int_{k}^{k'} \left(v_k(C(k'),\kappa,q(k')) - v_k(C(\kappa),\kappa,q(\kappa))\right) d\kappa \geq 0 \quad \forall\, k$ *and* k'.

PROOF: We shall first establish necessity and show that the incentive compatibility constraints given in equation (2) imply the two conditions of the lemma. By definition $r(k) = t(k) + C(k) - v(C(k),k,q(k))$ and $r(k') = t(k') + C(k') - v(C(k'),k',q(k'))$. So by the incentive compatibility constraints of equation (2)

$$\begin{aligned} r(k) &\geq t(k') + C(k') - v(C(k'),k,q(k')) \\ r(k') &\geq t(k) + C(k) - v(C(k),k',q(k)). \end{aligned}$$

Combining the definitions with the incentive constraints gives

$$-\left(v(C(k'),k',q(k')) - v(C(k'),k,q(k'))\right) \geq r(k') - r(k) \geq -\left(v(C(k),k',q(k)) - v(C(k),k,q(k))\right).$$

Hence by the continuity of $v(C,k,q)$ and the connectedness of the sets $\mathcal{C}$ (given the continuity of g) and $\mathcal{Q}$, the intermediate value theorem guarantees that this can be satisfied if and only if there are values $\bar{C}(k',k)$ and $\bar{q}(k',k)$ such that

$$r(k') - r(k) = -\left(v(\bar{C}(k',k),k',\bar{q}(k',k)) - v(\bar{C}(k',k),k,\bar{q}(k',k))\right) = -\int_{k}^{k'} v_k(\bar{C}(k',k),\kappa,\bar{q}(k',k))\,d\kappa.$$

Suppose without loss of generality that $k' > k$. Then using the Mean Value Theorem there is some $\hat{k} \in [k,k']$ such that

$$\frac{r(k') - r(k)}{k' - k} = v_k(\bar{C}(k',k),\hat{k},\bar{q}(k',k)). \tag{A.1}$$

Then by compactness we can find convergent subsequences such that $\bar{C}(k',k) \to \bar{C}(k)$ and $\bar{q}(k',k) \to \bar{q}(k)$ as $k' \to k$ so that taking the limit as $k' \to k$ gives

$$\dot{r}(k) = -v_k(\bar{C}(k),k,\bar{q}(k))$$

wherever this is defined. Then using equation (A.1) and taking the absolute value shows that

$$|r(k') - r(k)| \leq v_k(\bar{C}(k',k), \hat{k}, \bar{q}(k',k)|k' - k| \leq M|k' - k|$$

for some finite number M since v_k is assumed continuous on a closed and bounded interval and hence itself bounded. Hence $r(k)$ is Lipschtz continuous and hence absolutely continuous. Thus we can use the fundamental theorem of calculus and write

$$r(k) = r(\underline{k}) + \int_{\underline{k}}^{k} \dot{r}(\kappa)\, d\kappa. \tag{A.2}$$

Equally using the definition of $r(k',k)$,

$$r(k',k) = r(k') + v((C(k'),k',q(k')) - v(C(k'),k,q(k')).$$

Hence using equation (A.2) we have

$$\begin{aligned} r(k) - r(k',k) = r(k) - r(k') &- \int_{k}^{k'} v_k(C(k'),\kappa,q(k'))\, d\kappa \\ &- \int_{\underline{k}}^{k} v_k(C(\kappa),\kappa,q(\kappa))\, d\kappa + \int_{\underline{k}}^{k'} v_k(C(\kappa),\kappa,q(\kappa))\, d\kappa \\ &- \int_{k}^{k'} v_k(C(k'),\kappa,q(k'))\, d\kappa \\ = &- \int_{k}^{k'} \big(v_k(C(k'),\kappa,q(k')) - v_k(C(\kappa),\kappa,q(\kappa))\big)\, d\kappa \end{aligned}$$

Then since incentive compatibility requires $r(k) - r(k',k) \geq 0$ for all k and k' we have the second condition of the Lemma. Now consider sufficiency. It is obvious from the above equation that if the second condition of the Lemma is satisfied then $r(k) - r(k',k) \geq 0$ for all k and k' and hence it is incentive compatible. Equally if Assumption 4 is satisfied then $r(k)$ is incentive compatible if it satisfies the integral condition. From the single crossing property it is always possible to find some (C_1, q_1) and (C_2, q_2) such that

$$v_k(C_1,k,q_1) \leq v_k(C,k,q) \leq v_k(C_2,k,q_2)$$

for all k, C, q. Then since the integral condition is satisfied

$$\int_{k}^{k'} v_k(C_1,\kappa,q_1)\, d\kappa \leq \int_{k}^{k'} v_k(C,\kappa,q)\, d\kappa \leq \int_{k}^{k'} v_k(C_2,\kappa,q_2)\, d\kappa$$

or

$$v(C_1,k',q_1) - v(C_1,k,q_1) \leq r(k') - r(k) \leq v(C_2,k',q_2) - v(C_2,k,q_2)$$

where C is chosen equal to $\bar{C}(k',k)$ and $q = \bar{q}(k',k)$.

$$r(k') - r(k) = v(\bar{C}(k',k),k',\bar{q}(k',k)) - v(\bar{C}(k',k),k,\bar{q}(k',k)).$$

so that reversing the argument given above $r(k)$ is incentive compatible. □

PROPOSITION 4: *In any equilibrium in which the firm adopts a mixed strategy there are no mass points*

PROOF: Suppose there is a mass point at k_i. Let the mass at this point be f_d where $F(k_i-) + f_d = F(k_i)$. We shall define a new distribution $F_\epsilon(k)$ such that

$$F_\epsilon(k) = \begin{cases} F(k) + \frac{f_d}{\epsilon}(k - (k_i + \epsilon)) & \text{if } k \in [k_i, k_i + \epsilon] \\ F(k) & \text{otherwise} \end{cases}$$

where $\epsilon > 0$. We can choose ϵ small so that $F(k)$ is continuous on $[k, k + \epsilon]$. Note the $F_\epsilon(k_i) = F(k_i) - f_d = F(k_i-)$ and $F_\epsilon(k_i + \epsilon) = F(k_i + \epsilon)$. Hence $(1 - F_\epsilon(k)) - (1 - F(k)) = -\frac{f_d}{\epsilon}(k - (k_i + \epsilon))$ on $[k, k + \epsilon]$. Also since F is continuous on $(k_i, k_i + \epsilon]$ we have $dF_\epsilon(k) = dF(k) + \frac{f_d}{\epsilon}$ on this interval. Consider then a change in the distribution from $F(k)$ to $F_\epsilon(k)$, but keeping the contract unchanged. The contract remains incentive compatible but may no longer be optimal. Using equation (10) the effect of such a change in distribution on total welfare is

$$\int_{k_i}^{k_i+\epsilon} \omega(C(k), k, q(k)) \, (dF_\epsilon(k) - dF(k)) - \omega(C(k_i), k_i, q(k_i)) f_d \\ - (1 + \lambda)\rho \int_{k_i}^{k_i+\epsilon} ((1 - F_\epsilon(k)) - (1 - F(k))) \, dk.$$

Using the definition for F_ϵ this can be rewritten as

$$\text{(A.3)} \quad \frac{f_d}{\epsilon} \left(\int_{k_i}^{k_i+\epsilon} \{\omega(C(k), k, q(k)) - \omega(C(k_i), k_i, q(k_i)) + (1 + \lambda)\rho(k - (k_i + \epsilon))\} \; dk \right).$$

For notational simplicity write $C = C(k)$, $C_i = C(k_i)$ and so on. Decomposing the change in ω we have

$$\text{(A.4)} \quad \omega(C, k, q) - \omega(C_i, k_i, q_i) = (\omega(C, k, q) - \omega(C_i, k, q_i)) + (\omega(C_i, k, q_i) - \omega(C_i, k_i, q_i)).$$

By Assumption 5, ω is strictly concave in C and q and hence since it is differentiable

$$\omega(C, k, q) - \omega(C_i, k, q_i) > -\omega_C(C, k, q)(C_i - C) - \omega_q(C, k, q)(q_i - q).$$

The from the first-order conditions of equations (7) and (8) we have

$$-\omega_C(C, k, q)(C_i - C) - \omega_q(C, k, q)(q_i - q) = \varphi(k)(v_{kC}(C, k, q)(C_i - C) + v_{kq}(C, k, q)(q_i - q))$$

for some constant $\varphi(k) > 0$. Next using the strict concavity and differentiability of v_k

$$v_{kC}(C, k, q)(C_i - C) + v_{kq}(C, k, q)(q_i - q) > v_k(C_i, k, q_i) - v_k(C, k, q).$$

However by equation (9) $v_k(C, k, q) = v_k(C_i, k_i, q_i)$ and hence

$$v_k(C_i, k, q_i) - v_k(C, k, q) = v_k(C_i, k, q_i) - v_k(C_i, k_i, q_i).$$

Hence

$$\omega(C, k, q) - \omega(C_i, k, q_i) > \varphi(k) \, (v_k(C_i, k, q_i) - v_k(C_i, k_i, q_i)) \, .$$

Since we also have

$$\omega(C_i, k, q_i) - \omega(C_i, k_i, q_i) = -(1+\lambda)\left(v(C_i, k, q_i) - v(C_i, k_i, q_i)\right)$$

we can rewrite equation (A.4) as the inequality

$$\omega(C,k,q) - \omega(C_i,k_i,q_i) > \varphi(k)\left(v_k(C_i,k,q_i) - v_k(C_i,k_i,q_i)\right) - (1+\lambda)\left(v(C_i,k,q_i) - v(C_i,k_i,q_i)\right).$$

We can then use the mean value theorem to find values k^1 and k^2 between k_i and k so that

$$\begin{aligned}\omega(C,k,q) - \omega(C_i,k_i,q_i) > &- (1+\lambda)v_k(C_i,k_i,q_i)(k-k_i) + \varphi(k)v_{kk}(C_i,k^1,q_i)(k-k_i)\\ &- (1+\lambda)(v_k(C_i,k^2,q_i) - v_k(C_i,k_i,q_i))(k-k_i)\end{aligned}$$

where $(1+\lambda)v_k(C_i,k_i,q_i)(k-k_i)$ has been added and subtracted to the right-hand-side of the equation. Applying the mean value theorem once again we can find a value k^3 between k^2 and k_i and using $-v_k(C_i,k_i,q_i) = \rho$ the inequality becomes

$$\begin{aligned}\omega(C,k,q) - \omega(C_i,k_i,q_i) > &(1+\lambda)\rho(k-k_i) + \varphi(k)v_{kk}(C_i,k^1,q_i)(k-k_i)\\ &- (1+\lambda)v_{kk}(C_i,k^3,q_i)(k-k_i)(k^2-k_i)\end{aligned}$$

Taking $k > k_i$ and diving through by $k - k_i$ shows that we can find some $\epsilon > 0$ such that with $k - k_i < \epsilon$,

$$\omega(C,k,q) - \omega(C_i,k_i,q_i) > (1+\lambda)\rho(k-k_i).$$

Substituting into equation (A.3) and integrating then shows that the change in the welfare is positive. Since this improvement is achieved without changing the contract as the distribution is changed it is clear that the optimal contract in the new distribution cannot lower welfare and hence the original distribution could not have been an equilibrium. Finally note that the above arguments rules out mass points by changing the distribution to the right of the mass point and therefore does not work if there is a mass point at k^*. However if there is a mass of f_* at k^* a similar argument works be defining a distribution

$$F_\epsilon(k) = \begin{cases} F(k) + \frac{f_*}{\epsilon}(k - (k^* - \epsilon)) & \text{if } k \in [k^* - \epsilon, k^*] \\ F(k) & \text{otherwise} \end{cases}$$

which smooths out the mass point to the left. Reapplying the same argument as above also shows that such a change increases welfare and therefore the original distribution with mass at k^* could not have been an equilibrium. □

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