

# CURRENCY AREAS AND INTERNATIONAL ASSISTANCE\*

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## Abstract

This paper considers a simple stochastic model of international trade with three countries. Two of the three countries are in an economic union. Comparisons are made between equilibrium welfare for these two countries under fixed and flexible exchange rate regimes. Within the model it is shown that flexible exchange rate regimes generate greater welfare. However, we then consider comparisons of welfare when the two countries also engage in some international assistance in order to share risk. Such risk-sharing is limited by enforcement constraints of cross border assistance. It is shown that taking into account limited commitment risk-sharing fixed exchange rates or currency areas can dominate flexible exchange rate regimes reversing the previous result.

KEYWORDS: Monetary Union; Currency Areas; Fiscal Federalism; Limited Commitment; Mutual Insurance

JEL CODES: F12; F15; F31; F33.

## 1. INTRODUCTION

The present paper contributes to the theories of optimal currency areas and fiscal federalism. In an often cited paper Mundell (1961) argues that business cycles should be sufficiently (positively) correlated for a common currency area to be optimal. Bayoumi (1994) formally shows that negative shock correlation makes currency unions less desirable. As a consequence, the discussion about the UK's (non-)integration in the Euro currency area has often been driven by the fact that UK business cycles are not well correlated with continental Europe. Equally the discussion of when the accession countries should join the Euro currency area has been dominated by the debate of the congruence of the economic cycles of the accession countries and the rest of Europe.

One aspect of monetary integration that has received relatively little attention is the interaction of international assistance and business cycles on the optimality of currency areas. This is an important consideration as transfers between regions can be used as a means of insurance against regional income shocks as has been argued by Drèze (2000).<sup>1</sup> Moreover in the case of EU,

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\*Preliminary and incomplete. Please do not quote or use without permission. We thank Jacques Drèze for helpful comments. All errors are ours.

<sup>1</sup>The theory of fiscal federalism suggests that positive transfers between regions may take place mainly because of sharing of a public good, such as security (see also Alesina et al. 1995, Persson and Tabellini 1996), rather than sharing of risk.

shocks have been shown to be large (Forni and Reichlin 1999) and French and Poterba (1991) and Baxter and Jermann (1997) have shown that agents' risk diversification is insufficient suggesting that there are welfare gains if more insurance between regions can be agreed. Although inter-regional transfers in the EU are currently quite small (less than 1% GDP) they do increasingly depend on local income situations (EU Objective 1 Funds) and increasing integration is likely to mean that they take a more prominent role in smoothing interregional fluctuations. Thus it is important to examine the interaction between these risk-sharing measures and the optimality of a currency union.

In this paper, we model a situation where there are nominal wage and price rigidities. These wage and price rigidities would normally mean that there is a need for exchange rate flexibility and hence monetary union is inefficient. However, if the importance of transfers between countries to offset regional shocks is taken into account this result might be reversed. We model transfers between countries by assuming that they are limited by self-enforcing constraints. This is natural as there is no supra-national legal authority to enforce them. Hence countries will only make such transfers if they are in their own long-term interest. This long-term interest will be determined by the future benefits of risk sharing and by the punishment imposed for not making the requisite transfer. We shall assume that this punishment is that the country will not subsequently receive future assistance. Since the inefficiencies caused by nominal wage and price rigidities may increase the variance of regional incomes in currency unions, they may make it easier to enforce transfers because there is an increased punishment threat or sanction that can be applied to any country not making a transfer that is called upon to do so. Countries may thus prefer a monetary union because transfers are more likely to be sustainable or more insurance can be sustained. That is the benefit of insurance may outweigh the inefficiency cost of price rigidities. We shall show that this is indeed possible and that there exists a set of parameter values such that a monetary union is preferred to the flexible exchange rate system. As our model is highly stylized the importance of the paper is not in showing for what set of parameter values a currency union is optimal but in showing that the issue of currency integration cannot be divorced from the issue of fiscal integration through regional risk sharing.

The paper is organized as it follows. Section 2 presents the model and studies the equilibria under common currency area and flexible exchange rates. In Section 3 we study an economy with negatively correlated shocks and show when transfers between countries can be sustained and when a monetary union will deliver higher welfare than a flexible exchange rate system. Section 4 considers extensions to the basic model. These extensions quantitatively modify but do not qualitatively change the main results that the common currency system sustains greater levels of transfers and may therefore lead to higher welfare for certain parameter values. Section 5 concludes.

2. BASIC MODEL

We consider a simple model of international trade with three countries  $r = H, F, W$  (Home, Foreign and the rest of the World). We consider that Home and Foreign have close economic ties or are in an economic union and shall be interested in whether Home and Foreign should operate a currency union or retain a flexible exchange rate regime. This is the situation of the UK or accession countries and the Euro area countries or between Canada or Mexico and the US. In order to make the comparison between fixed and flexible exchange rates non-trivial we shall assume that there are some market imperfections in the labor and product markets. Following much of the literature on currency areas we shall assume that labor is immobile across countries and that the labor market is subject to nominal rigidities. However, we shall assume there is free trade between countries and in particular there are no transport or transactions costs to trade.<sup>2</sup> Further we shall assume that the goods markets in the Home and Foreign countries is monopolistically competitive with a continuum of varieties produced and price setting by firms. In the rest of the world firms produce a homogenous good which we label  $W$ .

There is a unit mass of consumer-workers in country  $H$  and the same mass in country  $F$ . In country  $W$  as a mass of  $X$  consumer-workers.<sup>3</sup> Labor is supplied inelastically. We shall assume that goods are non-storable and that consumers consume both domestically produced and foreign produced goods. We shall index goods by  $\zeta$  or  $\xi$  and let  $d_{rs}(\xi)$  denote the demand for good of variety  $\xi$  from consumers in country  $r$  which is produced in country  $s$ . Similarly we let  $d_{rW}$  denote the demand for good  $W$  in country  $r$ . Individuals in all countries have the same Dixit and Stiglitz (1977) type preference function given by

$$(1) \quad V(C_r) = V \left( \left( \int_0^1 d_{rr}(\zeta)^{\frac{\sigma-1}{\sigma}} d\zeta + \int_0^1 d_{rs}(\xi)^{\frac{\sigma-1}{\sigma}} d\xi \right)^{\frac{\sigma\mu}{\sigma-1}} \right) d_{rW}^{1-\mu},$$

where  $V$  is strictly concave,<sup>4</sup>  $C_r$  is the aggregate composite good in country  $r$  and  $\sigma > 1$  is the elasticity of substitution. The elasticity  $\sigma$  will be a measure of the degree of competition between firms and the higher is  $\sigma$  the greater will be the degree of competition between firms across countries. The parameter  $\mu$  measures the relative importance of the rest of the World. If  $\mu$  is close to one then the rest of the World has little importance and we are left with a two country model. Letting  $p_{rs}(\zeta)$  denote the price of good  $\zeta$  sold in country  $r$  and produced in country  $s$ , the budget constraint for the representative agent in country  $r$  is

$$\int_0^1 d_{rH}(\zeta) p_{rH}(\zeta) d\zeta + \int_0^1 d_{rF}(\xi) p_{rF}(\xi) d\xi + p_{rW} d_{rW} = Y_r + T_r$$

<sup>2</sup>Transport costs can be added to the model without substantially changing the conclusions of the paper. Without transport costs a currency union is never desirable without the insurance motives we model here. Thus introducing transportation costs will tend to strengthen our conclusions.

<sup>3</sup>Nothing will hinge on the relative size of  $X$ .

<sup>4</sup>For some of the subsequent analysis and the numerical calculations we shall assume that the utility function exhibits constant relative risk aversion preferences.

where  $Y_r$  is the income of consumers in country  $r$  and  $T_r$  is the transfer income received by country  $r$  expressed in the own countries currency. As we are examining an economic union between  $H$  and  $F$  we shall consider transfers only between the Home and Foreign country, that is  $T_W \equiv 0$ . With the utility function given in (1) the demand for good  $\zeta$  from consumers in country  $r$  which is produced in country  $s$  is given by

$$(2) \quad d_{rs}(\zeta) = \mu \frac{p_{rs}(\zeta)^{-\sigma} (Y_r + T_r)}{P_r^{1-\sigma}}$$

and where  $P_r$  is a price index for goods produced in  $H$  and  $F$  and is given by

$$(3) \quad P_r = \left( \int_0^1 p_{rr}(\zeta)^{1-\sigma} d\zeta + \int_0^1 p_{rs}(\xi)^{1-\sigma} d\xi \right)^{\frac{1}{1-\sigma}}.$$

Likewise the demand for good  $W$  in country  $r$  is

$$d_{rW} = (1 - \mu) \frac{(Y_r + T_r)}{p_{rW}}$$

where  $p_{rW}$  is the price of good  $W$  in country  $r$ .

As is well-known the composite consumption is linear in income and simple calculation shows that

$$(4) \quad C_r = \mu^\mu (1 - \mu)^{1-\mu} \frac{Y_r + T_r}{P_r^\mu p_{rW}^{1-\mu}}.$$

It will be convenient to emphasize the dependence of the composite consumption on the transfer and write it as a function of the transfer received  $C_r(T)$ .

The rate of exchange between currencies in the two countries will depend on whether there is a single currency area or a flexible exchange rate. However, in either case since we have assumed there is free and frictionless trade

$$p_{rs} = \varepsilon_{rs} p_{ss} \quad \forall r \neq s$$

where  $\varepsilon_{rs}$  converts currency of country  $s$  into the currency of country  $r$ . Since trade is frictionless  $\varepsilon_{rs} = 1/\varepsilon_{sr}$ . With three countries there are effectively two independent exchange rates and we shall write  $\varepsilon$  for  $\varepsilon_{HF}$  and  $\eta$  for  $\varepsilon_{HW}$ . Thus  $\varepsilon_{FW} = \varepsilon/\eta$ . For convenience we shall also write  $p_r(\zeta)$  for  $p_{rr}(\zeta)$  and  $p_{WW} = p_W$ . Therefore  $p_{HF}(\zeta) = \varepsilon p_F(\zeta)$ ,  $p_{FH}(\xi) = p_H(\xi)/\varepsilon$  and  $p_{HW} = \eta p_W$ . Then it is easy to check from equation (3) that  $P_H = \varepsilon P_F$  and that  $p_{FW} = p_{HW}/\varepsilon$ . Since there are no transfer from outside  $H$  and  $F$  we shall have  $T_H = -\varepsilon T_F$ . In the case of a common currency area  $\varepsilon = 1$ .

We now consider firm behavior. We shall let  $Y = Y_H + \varepsilon Y_F + \eta Y_W$  denote total world income measured in the home currency.<sup>5</sup> For firms in industry  $\zeta$  in country  $H$  the demand they face from Home and other consumers is

$$\begin{aligned}
 d_H(\zeta) &= d_{HH}(\zeta) + d_{FH}(\zeta) + d_{WH}(\zeta) \\
 (5) \quad &= \mu \left( \frac{p_H(\zeta)^{-\sigma} (Y_H + T_H)}{P_H^{1-\sigma}} + \frac{\left(\frac{p_F(\zeta)}{\varepsilon}\right)^{-\sigma} (Y_F + T_F)}{P_F^{1-\sigma}} + \frac{\left(\frac{p_W}{\eta}\right)^{-\sigma} Y_W}{P_W^{1-\sigma}} \right) \\
 &= \mu \frac{p_H(\zeta)^{-\sigma}}{P_H^{1-\sigma}} Y
 \end{aligned}$$

where the second equality follows because  $P_H = \varepsilon P_F$  and  $T_H = -\varepsilon T_F$ . Similarly

$$(6) \quad d_F(\xi) = \mu \frac{p_H(\zeta)^{-\sigma}}{P_H^{1-\sigma}} \frac{Y}{\varepsilon} \quad \text{and} \quad d_R = (1 - \mu) \frac{1}{p_W} \frac{Y}{\eta}.$$

Firms take demands as given and costs are determined by labor requirements. Labor requirements are given by a fixed coefficient technology where to produce one unit of output in country  $r$  requires  $a_r$  units of labor. The price of labor is denoted by  $w_r$ . However, we shall assume that the labor market is imperfect and that nominal wages are fixed and therefore normalize  $w_r = 1$ .<sup>6</sup> With flexible exchange rates the exchange rates will adjust to achieve full employment in each country. However, if there is a common currency between countries  $H$  and  $F$ , the nominal rigidity in wages will cause unemployment in the relatively unproductive country. In Section 3 we shall assume that  $a_H$  and  $a_F$  are stochastic and specify a simple stochastic process. However, because input decisions will be made once  $a_r$  is known and since there are no intertemporal linkages in production or consumption we can determine equilibria as if  $a_r$  were fixed and given. In country  $W$ ,  $a_W = 1$  and we assume that firms are perfectly competitive. Thus we normalize so that  $p_W = 1$  which in turn implies  $p_{HW} = \eta$  and  $P_{FW} = \eta/\varepsilon$ . In countries  $H$  and  $F$  firms are assumed to be monopolistically competitive and will set prices to maximize profits given wage costs. Given the fixed costs and the iso-elastic demand functions given in equation (2), prices are set at a mark-up over cost

$$(7) \quad p_r(\zeta) = \frac{\sigma}{\sigma - 1} a_r = \frac{a_r}{\rho}, \quad r \in \{H, F\}$$

where  $\rho \equiv (\sigma - 1)/\sigma$ . It follows from equation (7) that all firms in countries  $H$  and  $F$  set the same price which is simply a mark-up over costs. This mark-up depends on the elasticity of substitution between commodities and when substitution is perfect ( $\sigma = \infty$  or  $\rho = 1$ ) there is perfect competition and price equal marginal cost. Equally it follows from equation (7) that demand is the same for all goods  $d_r(\zeta) = d_r$  and so all industries in a given country are of equal size. Thus

<sup>5</sup>Note that transfers do not affect world demand.

<sup>6</sup>Assuming that nominal wages are completely inflexible is obviously an extreme assumption which we make for convenience only. As argued by Mundell (1961) this a priori makes a currency union less desirable.

total labor demand is simply  $\ell_r = a_r d_r$ . By assumption all profits accrue to firms owners and for convenience we assume that all profits are spent locally. Thus the national income in country  $r$  is  $Y_r = p_r d_r = p_r \ell_r / a_r$ . From equation (7) we may therefore write this national income as

$$(8) \quad Y_H = \frac{\ell_H}{\rho} \quad \text{and} \quad Y_F = \frac{\ell_F}{\rho}.$$

Also since  $p_W = 1$ ,  $\ell_W = X$  and  $a_W = 1$ , national income in  $W$  is  $Y_W = X$ . Equally the price index of equation (3) can be re-written using equation (7) as

$$P_H = (p_H^{1-\sigma} + \varepsilon^{1-\sigma} p_F^{1-\sigma})^{\frac{1}{1-\sigma}} = \frac{1}{\rho} (a_H^{1-\sigma} + \varepsilon^{1-\sigma} a_F^{1-\sigma})^{\frac{1}{1-\sigma}} = \varepsilon P_F.$$

An equilibrium is then a set of prices and demands such that (i) consumers maximize their utility given their budget constraint and given prices, (ii) firms set their profit maximizing prices and (iii) product markets clear. Since the labor market is imperfect in countries  $H$  and  $F$  we do not assume that the labor market clears and there may be some situations, discussed below, where labor markets do not clear. In either case the product demand equal product supply conditions are given from equations (5) and (6) and the fixed coefficient technology as

$$(9) \quad \mu \frac{p_H(i)^{-\sigma} Y}{P_H^{1-\sigma}} = \frac{\ell_H}{a_H} \quad \text{and} \quad \mu \frac{p_F(i)^{-\sigma} \left(\frac{Y}{\varepsilon}\right)}{P_F^{1-\sigma}} = \frac{\ell_F}{a_F}$$

together with the condition  $(1 - \mu)(Y/\eta) = X$ . Using the fact that  $P_H = \varepsilon P_F$  and  $p_H/p_F = a_H/a_F$  and taking the ratio of the two equations in (9) gives

$$(10) \quad \varepsilon = \left(\frac{a_H}{a_F}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\ell_H}{\ell_F}\right)^{\frac{1}{\sigma}} \quad \text{and} \quad \eta = \frac{(1-\mu)}{\rho \mu X} (\ell_H + \varepsilon \ell_F).$$

These conditions will determine either the exchange rates  $\varepsilon$  and  $\eta$  or if  $\varepsilon$  is fixed the first condition determines level of employment in either country  $H$  or  $F$ .

### 2.1. Common currency area

We now consider the equilibrium in which countries  $H$  and  $F$  form a common currency area so that  $\varepsilon = 1$ . In this case the price index is the same in both countries,  $P_H = P_F$  and we let  $P$  denote this common index. Then

$$P = (p_H^{1-\sigma} + p_F^{1-\sigma})^{\frac{1}{1-\sigma}} = \frac{1}{\rho} (a_H^{1-\sigma} + a_F^{1-\sigma})^{\frac{1}{1-\sigma}}.$$

There are two cases to consider depending on the sign of  $a_H - a_F$ . If  $a_H < a_F$  the Home country is relatively more productive and requires less labor to produce any given quantity of output. With this parametrization and fixed wages it is easy to check that a share of country  $F$ 's labor force will be unemployed,  $\ell_F < 1$ , as its costs will be higher and hence demand will fall. In contrast

there will be full employment,  $\ell_H = 1$ , in country  $H$ . It then follows from equation (10) that  $\ell_F = (a_H/a_F)^{\sigma-1} < 1$ . If on the other hand  $a_H > a_F$  then there will be unemployment in the Home country and  $\ell_H = (a_H/a_F)^{1-\sigma} < 1 = \ell_F$ . Using equation (4) and the price index  $P$  given by equation (2.1), the composite consumption in each of the two countries is given by

$$\begin{aligned} C_H^c(T) &= \mu X^{1-\mu} (\ell_H + \rho T) (\ell_H + \ell_F) (a_H^{1-\sigma} + a_F^{1-\sigma})^{\frac{\mu}{\sigma-1}} \\ C_F^c(T) &= \mu X^{1-\mu} (\ell_F + \rho T) (\ell_H + \ell_F) (a_H^{1-\sigma} + a_F^{1-\sigma})^{\frac{\mu}{\sigma-1}} \end{aligned}$$

where the superscript  $c$  denotes that the consumption is under a currency union and where the employment levels are determined by Hence the level of employment in the two countries is given by

$$\ell_H = \min \left\{ \left( \frac{a_H}{a_F} \right)^{1-\sigma}, 1 \right\} \quad \text{and} \quad \ell_F = \min \left\{ \left( \frac{a_H}{a_F} \right)^{\sigma-1}, 1 \right\}.$$

## 2.2. Flexible exchange rate system

We now consider equilibrium output and consumption under a flexible exchange rate system. The introduction of an exchange rate provides an additional instrument to allow relative prices to alter and production and employment to increase. Although nominal wages are fixed at  $w_H = w_F = 1$  the exchange rate allows the relative wages  $\varepsilon w_F/w_H$  to adapt. Under a flexible exchange rate system there is no real wage rigidity so that labor market clears in the both the high and low productivity countries, i.e.  $\ell_H = \ell_F = 1$ . Then equilibrium output in both  $H$  and  $F$  is  $Y_H = Y_F = 1/\rho$  and the exchange rates are determined from (10):

$$(11) \quad \varepsilon = \left( \frac{a_H}{a_F} \right)^{\frac{\sigma-1}{\sigma}} \quad \text{and} \quad \eta = \frac{(1-\mu)(1+\varepsilon)}{\rho\mu X}.$$

If  $a_H < a_F$  then country  $F$  has relatively low productivity and its currency depreciates making its exports relatively cheaper to the Home country. Likewise if  $a_H > a_F$  then country  $F$  is relatively more productive and its currency will appreciate so that its exports become relatively more expensive to the Home country. With this exchange rate adjustment and remembering that  $P_H = \varepsilon P_F$ , the composite consumption in each country is given by from equation (4) as

$$\begin{aligned} C_H^f(T) &= \mu X^{1-\mu} (1 + \rho T) (1 + \varepsilon)^{\mu-1} (a_H^{1-\sigma} + \varepsilon^{1-\sigma} a_F^{1-\sigma})^{\frac{\mu}{\sigma-1}} \\ C_F^f(T) &= \mu X^{1-\mu} \varepsilon (1 + \rho T) (1 + \varepsilon)^{\mu-1} (a_H^{1-\sigma} + \varepsilon^{1-\sigma} a_F^{1-\sigma})^{\frac{\mu}{\sigma-1}} \end{aligned}$$

where the superscript  $f$  indicates that the consumption is calculated under flexible exchange rates and where the exchange rate  $\varepsilon$  is as given in equation (11).

In the absence of transfers, an appropriate exchange rate policy (common currency or flexible exchange rate) also allows countries to achieve the highest aggregate consumption. One readily shows that  $C_r^f(0) \geq C_r^c(0)$  for  $r = H, F$ . Thus we have the standard result

PROPOSITION 1: *Under autarchy ( $T_r = 0$ ), aggregate consumption at any state is higher under flexible exchange rate system:  $C_r^f(0) \geq C_r^c(0)$ ,  $r = H, F$ .*

PROOF: Take the case  $a_H < a_F$ . We have

$$\begin{aligned} C_H^f(0) &= \mu X^{1-\mu} (a_H^{1-\sigma} + \varepsilon^{1-\sigma} a_F^{1-\sigma})^{\frac{\mu}{\sigma-1}} (1 + \varepsilon)^{\mu-1} \\ C_H^c(0) &= \mu X^{1-\mu} (a_H^{1-\sigma} + a_F^{1-\sigma})^{\frac{\mu}{\sigma-1}} \left( 1 + \left( \frac{a_H}{a_F} \right)^{\sigma-1} \right)^{\mu-1}. \end{aligned}$$

Since  $a_H < a_F$ ,  $\varepsilon^{1-\sigma} > 1$  and  $\varepsilon > (a_H/a_F)^{\sigma-1}$ . Hence  $C_H^f(0) > C_H^c(0)$ . Also

$$\begin{aligned} C_F^f(0) &= \varepsilon \mu X^{1-\mu} (a_H^{1-\sigma} + \varepsilon^{1-\sigma} a_F^{1-\sigma})^{\frac{\mu}{\sigma-1}} (1 + \varepsilon)^{\mu-1} \\ C_F^c(0) &= \mu X^{1-\mu} \left( \frac{a_H}{a_F} \right)^{\sigma-1} (a_H^{1-\sigma} + a_F^{1-\sigma})^{\frac{\mu}{\sigma-1}} \left( 1 + \left( \frac{a_H}{a_F} \right)^{\sigma-1} \right)^{\mu-1}. \end{aligned}$$

Since  $a_H < a_F$ ,  $\varepsilon > (a_H/a_F)^{\sigma-1}$  and this together with the fact that  $\varepsilon^{1-\sigma} > 1$  shows that  $C_F^f(0) > C_F^c(0)$ . A similar argument applies in the case where  $a_H > a_F$ .  $\square$

Thus in this model if there are no transfers between countries  $H$  and  $F$  then a common currency area will always be dominated by a flexible exchange rate regime.

### 3. CURRENCY AREAS UNDER NEGATIVELY CORRELATED SHOCKS

We shall now consider transfers between countries  $H$  and  $F$  and how this affects the result that a flexible exchange rate regime dominates a common currency area that we have seen in Proposition 1. To do this we will need to specify a stochastic process for productivity shocks so that there are some potential mutual gains to transfers. The role of shocks has often played a key role in debates about common currency areas. Taking into account transactions costs Mundell (1961) argues that business cycles should be sufficiently (positively) correlated for a common currency area to be optimal. Bayoumi (1994) formally shows that negative correlation of shocks makes currency unions less desirable. As a consequence, the discussion about the UK's (non-)integration in the Euro currency area has often been driven by the fact that UK business cycles are not well correlated with continental Europe.

In this section we shall show that the conclusion of Mundell (1961) can be reversed. That is we shall show that a common currency area can be optimal when shocks are negatively correlated. To do this we take the extreme assumption that shocks are perfectly negatively correlated. Thus we will make assumptions, absence of transactions costs and negatively correlated shocks, that are usually seen as inimical to currency unions, yet show that a common currency area can be optimal in these circumstances. Our argument lies in the fact that when there are productivity shocks and income variability across countries it will be desirable to try and smooth these shocks by making transfers between countries. However, transfers across countries may be difficult to

legally enforce and therefore such insurance transfers must be self-enforcing and enforcement may sometimes be helped by the threat of a more severe punishment.

To proceed we shall assume that the technology in countries  $H$  and  $F$  is stochastic. We shall assume there are just two symmetric states, so that the two countries either have a good or a bad productivity shock. Further we suppose that these states are perfectly negatively correlated states so that the labor requirements are  $(a_H, a_F) = (z, 1)$  in state 1 and  $(a_H, a_F) = (1, z)$  in state 2. We assume that  $z < 1$  so that the Home country has a good productivity shock is state 1 and the Foreign country has the good productivity shock in state 2. To preserve symmetry we assume that each state occurs with equal probability. With these assumptions, the aggregate composite consumption under the common currency in the two states is given by

$$\begin{aligned} C_G^c(T) &= \mu X^{1-\mu} (1 + \rho T) z^{(\mu-1)(\sigma-1)} (1 + z^{1-\sigma})^{\frac{\sigma(\mu-1)+1}{\sigma-1}} \\ C_B^c(T) &= \mu X^{1-\mu} (z^{\sigma-1} + \rho T) z^{(\mu-1)(\sigma-1)} (1 + z^{1-\sigma})^{\frac{\sigma(\mu-1)+1}{\sigma-1}} \end{aligned}$$

where  $C_G^c(T)$  is the consumption of the Home (Foreign) country in state 1 (2) when it experiences a good productivity shock and  $C_B^c(T)$  is the consumption of the Home (Foreign) country in state 2 (1) when it experiences a bad productivity shock. The transfer  $T$  is the amount received and expressed in the countries own currency. Since  $z^{\sigma-1} < 1$  it can be seen that absent any transfers the Home (Foreign) country has higher consumption in state 1 (2), when it experiences a positive productivity shock.

Under the flexible exchange rate regime the exchange rate in state 1 is  $\varepsilon_1 = z^{\frac{\sigma-1}{\sigma}} < 1$  as the Foreign currency depreciates when the Home country has a positive productivity shock. In contrast, in state 2 when the Foreign country has a positive productivity shock its exchange rate appreciates to  $\varepsilon_2 = z^{\frac{1-\sigma}{\sigma}} > 1$ . Then calculating the aggregate composite consumption in each state for the two countries gives:

$$\begin{aligned} C_G^f(T) &= \mu X^{1-\mu} (1 + \rho T) z^{\frac{1-\sigma}{\sigma}} \left(1 + z^{\frac{1-\sigma}{\sigma}}\right)^{\frac{\sigma(\mu-1)+1}{\sigma-1}} \\ C_B^f(T) &= \mu X^{1-\mu} (1 + \rho T) \left(1 + z^{\frac{1-\sigma}{\sigma}}\right)^{\frac{\sigma(\mu-1)+1}{\sigma-1}}. \end{aligned}$$

As  $z^{\frac{1-\sigma}{\sigma}} > 1$  it can be seen that absent any transfers the Home (Foreign) country has higher consumption in state 1 (2) when it has positive productivity shock than in state 2 (1), i.e.  $C_G^f(0) > C_B^f(0)$ .

### 3.1. First-best transfers

In this section we determine the first-best transfers and show that if the first-best transfers are implemented, then the flexible exchange rate regime yields greater welfare. The first-best transfers will equalize composite consumptions whether countries have good or bad productiv-

ity shocks. Let  $\tau_*^i$  denote the transfer received by the country with the bad productivity shock under regime  $i \in \{c, f\}$ . This will be the transfer received by the Home (Foreign) country in state 2 (1).

Under the common currency the transfer received by country with the bad productivity shock equals the transfer made by the country with the good productivity shock. Thus the first-best transfer can be found by solving  $C_G^c(-\tau) = C_B^c(\tau)$ . Calculating this transfer and the first-best composite consumption  $C_*^c$  gives

$$\tau_*^c = \frac{1}{2\rho} (1 - z^{\sigma-1}) \quad \text{and} \quad C_*^c = \frac{1}{2} \mu X^{1-\mu} z^{\mu(\sigma-1)} (1 + z^{1-\sigma})^{\frac{\mu\sigma}{\sigma-1}}.$$

It can be seen that since  $z^{\sigma-1} < 1$ , the transfer  $\tau_*^c$  is positive and decreasing in  $z$ . It can also be seen that in this simple model transfers are independent of the rest of the World for two reasons. First, the transfer has no effect on world income  $Y$  and secondly it does not depend on the degree of preference  $\mu$  for rest of the World goods.

Under flexible exchange rates if  $\tau_*^f$  is received by the country with the bad productivity shock then the transfer made by the country with the good productivity shock is  $z^{\frac{\sigma-1}{\sigma}} \tau_*^f$ . In state 1 (2) it is the Home (Foreign) country which has the good productivity shock so the transfer made is  $\varepsilon_1 \tau_*^f$  ( $\tau_*^f / \varepsilon_2$ ) where  $\varepsilon_1 = z^{\frac{\sigma-1}{\sigma}} = 1 / \varepsilon_2$ . The first-best transfer is therefore found by solving  $C_G^f(-z^{\frac{\sigma-1}{\sigma}} \tau_*^f) = C_B^f(\tau_*^f)$ . This gives the first-best transfer and consumption as given by

$$\tau_*^f = \frac{1}{2\rho} \left( z^{\frac{1-\sigma}{\sigma}} - 1 \right) \quad \text{and} \quad C_*^f = \frac{1}{2} \mu X^{1-\mu} \left( 1 + z^{\frac{1-\sigma}{\sigma}} \right)^{\frac{\mu\sigma}{\sigma-1}}.$$

Since  $z^{\frac{1-\sigma}{\sigma}} > 1$ , the transfer is positive and decreasing in  $z$ . Again this transfer does not affect  $Y$  and is independent of  $\mu$ .

It is natural to expect that lower consumption in autarchy implies lower consumption under full insurance and this is demonstrated in the next proposition.

**PROPOSITION 2:** *First best transfers yield higher aggregate utility under flexible exchange rate system:  $C_*^f \geq C_*^c$ .*

**PROOF:** Rewriting the formulas above for the composite consumptions so that they can be compared gives

$$C_*^c = \frac{1}{2} z^{-\mu} (1 + z^{\sigma-1})^{\frac{\mu\sigma}{\sigma-1}} \quad \text{and} \quad C_*^f = \frac{1}{2} z^{-\mu} \left( 1 + z^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\mu\sigma}{\sigma-1}}.$$

Since  $z^{\frac{\sigma-1}{\sigma}} > z^{\sigma-1}$  for  $z < 1$  and  $\sigma > 1$  it follows that  $C_*^f \geq C_*^c$ .  $\square$

Thus if it were possible to sustain the first-best transfers then again the common currency area would be sub-optimal. However, we shall be interested in cases where the common currency

regime sustains more insurance than the flexible exchange rate regime. For some parameter values however, the flexible exchange rate regime will always dominate as  $C_B^f(0) > C_*^c$ . That is the worst outcome in the flexible exchange rate regime delivers more welfare than the first-best welfare with a common currency. The next proposition gives parameter values such that the common currency can never be optimal.

PROPOSITION 3: *If  $z > (1/2)(1+z)^{(1-\mu)}$  then  $C_B^f(0) > C_*^c$ .*

PROOF: The composite consumptions  $C_B^f(0)$  and  $C_*^c$  are both functions of  $z$ ,  $\sigma$  and  $\mu$ . Therefore let  $g(z, \sigma, \mu) = C_B^f(0)/C_*^c$  denote the ratio of the composite consumptions. It can be checked that  $\lim_{\sigma \rightarrow 1} g(z, \sigma, \mu) = 1$  and that the function  $g(z, \sigma)$  asymptotes from above so that  $\lim_{\sigma \rightarrow \infty} g(z, \sigma, \mu) = 2z/(1+z)^{(1-\mu)}$ . Hence for values of  $z$  and  $\mu$  such that  $z > (1/2)(1+z)^{(1-\mu)}$ ,  $g(z, \sigma, \mu) \geq 1$  for all values of  $\sigma \in (1, \infty)$ .  $\square$

### 3.2. Sustainable Assistance

There may be many reasons why the first-best transfers between countries  $H$  and  $F$  cannot be achieved. We consider the case where the first-best transfers cannot be achieved because there is no supra-legal authority to enforce transfer across countries. To allow for the possibility of transfers we therefore consider that countries interact repeatedly over an infinite horizon. Since transfers cannot be legally enforced, countries can renege on any agreement if they find it in their interest not to make a transfer and hence any assistance programme has to be designed to be self-enforcing. Thomas and Worrall (1988) examine self-enforcing wage contracts between an employer and an employee and a similar approach can be applied here. Thus we presume that countries make a tacit agreement on a programme of mutual assistance and specify the transfers to be made by the country with the good productivity shock and to be received by the country with the bad productivity shock in every possible contingency. We shall assume that any breach of this tacit agreement results in a breakdown in which no transfers are made.<sup>7</sup>

To consider such self-enforcing transfers between countries  $H$  and  $F$  let  $h_t$  denote the history of good and bad outcomes for a particular country.<sup>8</sup> Let  $G^t$  denote the good productivity shock outcome at date  $t$  and  $B^t$  denote bad productivity shock outcome at date  $t$ . Then  $h_t$  is a list of  $G$ 's and  $B$ 's where  $h_0 = \emptyset$ . An assistance programme in regime  $i \in \{c, f\}$  then specifies a transfer  $\tau_B^i(h_{t-1})$  to be made to the country with the bad productivity shock if the previous history is  $h_{t-1}$  and the transfer to be made by the country with the good productivity shock  $\tau_G^i(h_{t-1})$ . The short-term loss to the country with a good productivity shock of making the required transfer at

<sup>7</sup>A breakdown in which no transfers are made is the worst possible outcome and is sub-game perfect. It is not however renegotiation-proof. Nevertheless it can be shown that in the current context replacing these punishments with one that are renegotiation-proof will not change the qualitative or quantitative properties of the assistance programme.

<sup>8</sup>As we have assumed that shocks are perfectly negatively correlated this history has only to be specified for one arbitrary country and is equivalent to specifying a history of states. We are also assuming that all other aspects of the economy are unchanging over time.

time  $t \geq 1$  relative to making no transfer is

$$V(C_G^i(-\tau_G^i(h_{t-1}))) - V(C_G^i(0)).$$

Likewise the short-term gain at date  $t \geq 1$  for the country receiving a transfer is

$$V(C_B^i(\tau_B^i(h_{t-1}))) - V(C_B^i(0)).$$

To evaluate future gains and losses we shall assume that countries discount the future by a common discount factor  $\delta \in (0, 1]$ . Then the discounted long-term gain from adhering to the agreed transfers from the next period is (discounted back to period  $t + 1$ )

$$E \left[ \sum_{j=0}^{\infty} \delta^j \left\{ \frac{1}{2} \left[ V(C_G^i(-\tau_G^i(h_{t+j}))) - V(C_G^i(0)) \right] + \frac{1}{2} \left[ V(C_B^i(\tau_B^i(h_{t+j}))) - V(C_B^i(0)) \right] \right\} \right].$$

where the expectation  $E$  is taken over all future histories of good and bad productivity shocks from date  $t$  onward,  $\tau_G^i(h_{t+j})$  is the transfer promised to be made by the country with the good productivity shock at date  $t + j + 1$  given that the history up to time  $t$  was  $h_t$  and  $\tau_B^i(h_{t+j})$  is the transfer to be received by the country with the bad productivity shock at date  $t + j + 1$  given that the history up to time  $t$  was  $h_t$ . Letting  $\mathcal{V}_G^i(h_t)$  denote the net discounted net utility from date  $t + 1$  for the country with the good productivity shock, i.e. where the history is  $h_{t+1} = (h_t, G^{t+1})$ , and  $\mathcal{V}_B^i(h_t)$  be the net utility for a country with a bad productivity shock, i.e. where the history is  $h_{t+1} = (h_t, B^{t+1})$ , we have the recursive equations

$$\begin{aligned} \mathcal{V}_G^i(h_t) &= V(C_G^i(-\tau_G^i(h_{t-1}))) - V(C_G^i(0)) + \delta \left[ \frac{1}{2} \mathcal{V}_G^i(h_t, G^{t+1}) + \frac{1}{2} \mathcal{V}_B^i(h_t, B^{t+1}) \right], \\ \mathcal{V}_B^i(h_t) &= V(C_B^i(\tau_B^i(h_{t+j}))) - V(C_B^i(0)) + \delta \left[ \frac{1}{2} \mathcal{V}_G^i(h_t, G^{t+1}) + \frac{1}{2} \mathcal{V}_B^i(h_t, B^{t+1}) \right]. \end{aligned}$$

A country in the economic union will make a transfer if the expected discounted risk-sharing benefits from future transfers exist the costs of making the current transfer. Since renegeing leads to exclusion, the discounted net utilities must be non-negative at every history

$$(12) \quad \mathcal{V}_G^i(h_t) \geq 0 \quad \text{and} \quad \mathcal{V}_B^i(h_t) \geq 0 \quad \forall h_t.$$

We shall say that an assistance programme is *sustainable* if equations (12) are satisfied. In addition we shall say that the assistance programme is *feasible* if  $\tau_G^c(h_t) = \tau_B^c(h_t)$  for all  $h_t$  under the common currency regime or  $\tau_G^f(h_t) = z^{\frac{\sigma-1}{\sigma}} \tau_B^f(h_t)$  under flexible exchange rates.

From Propositions 1 and 2 we know that if either first-best transfers or no transfers are sustainable then the flexible exchange rate dominates. However, we shall be interested in comparing welfare in cases where some insurance is possible (under the common currency regime) but where first-best transfers are not sustainable (under the flexible exchange rate regime). To make

CURRENCY AREAS

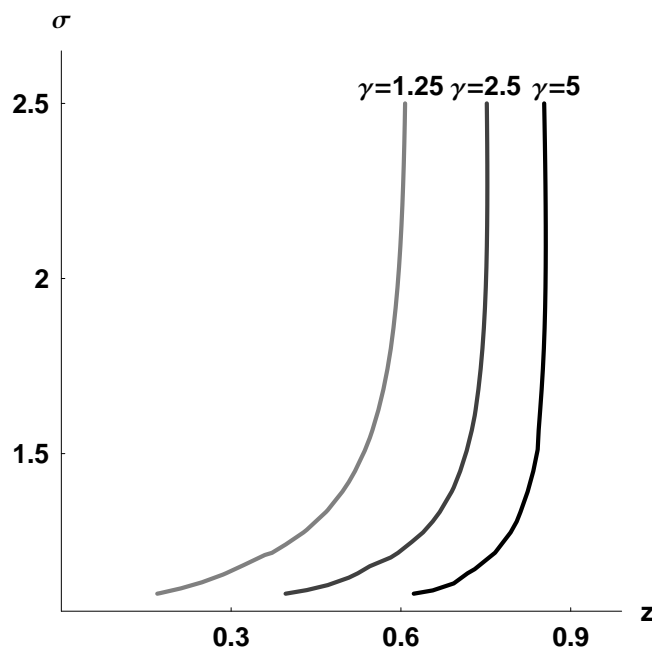


Figure 1: PARAMETER VALUES FOR WHICH FLEXIBLE EXCHANGE RATE ALWAYS DOMINATES.

such comparisons we shall assume that the utility function exhibits constant relative risk aversion preferences

$$V(C) = \begin{cases} \frac{C^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 1 \text{ and } \gamma \neq 1 \\ \ln(C) & \text{if } \gamma = 1 \end{cases}$$

where  $\gamma$  is the coefficient of relative risk aversion. We also know from Proposition 3 that welfare will only be higher under common currency areas for certain values of  $z$  and  $\sigma$ . Figure 1 illustrates regions of the parameter space for which flexible exchange rates always dominate because the welfare under flexible exchange rates with no transfers exceeds the welfare under common currency with first-best transfers. Figure 1 shows where flexible exchanges rates always dominate and where they may not for three different values of  $\gamma$  where  $\mu = 0.05$ . To the right of each line the flexible exchange rate regime will always dominate for the given value of  $\gamma$ .<sup>9</sup>

Having established parameter values such that the flexible exchange rate regime always dominates no matter what transfers are sustainable the next two subsections therefore consider for what parameter values the first-best transfers can be sustained and for what parameter values no transfers can be sustained. The next sub-section will then compare welfare in the two regimes.

<sup>9</sup>If countries are risk-neutral then the the evaluation is simply in terms of expected consumption and the flexible exchange rate regime always dominates.

### 3.3. Sustainable first best

In this subsection we consider the discount factors for which the first best-best transfers are sustainable. The first-best transfers are history independent and provide consumption of  $C_*^i$  in both countries. Because the first-best transfers are independent of history and because a country will only breach when it has a good productivity shock and is called upon to make a transfer, they are sustainable if and only if

$$V(C_*^i) - V(C_G^i(0)) + \frac{\delta}{(1-\delta)} \left\{ \frac{1}{2} [V(C_*^i) - V(C_G^i(0))] + \frac{1}{2} [V(C_*^i) - V(C_B^i(0))] \right\} \geq 0.$$

Rewriting this equation shows that these transfers are sustainable for discount factors above a critical level  $\bar{\delta}^i$  where

$$\bar{\delta}^i \equiv \frac{V(C_G^i(0)) - V(C_*^i)}{\frac{1}{2} [V(C_G^i(0)) - V(C_B^i(0))]}.$$

Making use of the constant relative risk aversion specification of preferences the critical values of the discount factors above which the first-best transfers are sustainable are independent of  $\mu$  and are given by

$$\bar{\delta}^f(z) = \bar{\delta}^c(z^{\frac{1}{\sigma}}) = \begin{cases} 2 \frac{1 - \left[ \frac{1}{2} \left( 1 + z^{\frac{\sigma-1}{\sigma}} \right) \right]^{1-\gamma}}{1 - z^{\frac{\sigma-1}{\sigma}(1-\gamma)}} & \text{if } \gamma > 1 \text{ and } \gamma \neq 1 \\ \frac{-\ln(2) + \ln(1 + z^{\frac{\sigma-1}{\sigma}})}{\ln(z^{\frac{\sigma-1}{\sigma}})} & \text{if } \gamma = 1. \end{cases}$$

One can show that for  $\gamma \geq 1$ ,  $\bar{\delta}^c(z)$  is a positive and increasing function of  $z$  with  $\bar{\delta}^c(0) = 0$  and  $\lim_{z \rightarrow 1} \bar{\delta}^c(z) = 0$ . Therefore as  $z < z^{1/\sigma} < 1$ , we get that  $\bar{\delta}^f(z) > \bar{\delta}^c(z)$ .

### 3.4. Non-sustainability of assistance

We now consider the case where the two countries  $H$  and  $F$  cannot sustain any transfers. We first define autarky ratio of marginal utilities as the ratio of the marginal utility of the country suffering the bad productivity shock to the marginal utility of the country experiencing the good productivity shock when no transfers are made. This is given by

$$RA^i(z) = \frac{dV(C_B^i(T))/dT}{dV(C_G^i(T))/dT} \Big|_{T=0}.$$

With constant relative risk aversion preferences we have

$$RA^f(z) = RA^c(z^{\frac{1}{\sigma}}) = z^{-\gamma\rho}$$

which is again independent of  $\mu$ . To find circumstances where no transfers are sustainable we need only consider some small and history independent transfers  $\tau_G^i$  and  $\tau_B^i$ . As a country will breach when it has a good productivity shock and is called upon to make a transfer, no transfer

is sustainable if

$$(13) \quad V(C_G^i(-\tau_G^i)) - V(C_G^i(0)) + \frac{\delta}{(1-\delta)} \left\{ \frac{1}{2} [V(C_G^i(-\tau_G^i)) - V(C_G^i(0))] + \frac{1}{2} [V(C_B^i(\tau_B^i)) - V(C_B^i(0))] \right\} < 0.$$

Approximating the left hand side around  $\tau_B^i = \tau_G^i = 0$  gives the condition

$$\left(1 - \frac{\delta}{2}\right) + \frac{\delta}{2} RA^i(z) < 0.$$

This gives a critical discount factor  $\underline{\delta}^i$  below which no transfers can be sustained in regime  $i \in \{c, f\}$ . Using the assumption of constant relative risk aversion preferences gives

$$\underline{\delta}^f = \underline{\delta}^c(z^{\frac{1}{\sigma}}) = \frac{2}{1 + z^{-\gamma\rho}}.$$

One readily can show that  $\underline{\delta}^c(z)$  is an increasing function of  $z$  with  $\lim_{z \rightarrow 0} \underline{\delta}^c(z) = 0$  and  $\underline{\delta}^c(1) = 1$ . Therefore  $\underline{\delta}^c < \underline{\delta}^f$ . This together with the condition of the previous subsection gives the following result.

**PROPOSITION 4:** *Common currency areas are more likely to sustain an equilibrium with first best transfers and less likely to sustain an equilibrium with zero transfers than flexible exchange rate systems.*

Figure 2 depicts the critical discount factors that sustain first best and autarchy for given values of  $\sigma$ ,  $\gamma$  and  $\mu$ . Below the line  $\underline{\delta}^i$  no transfers can be sustained in regime  $i \in \{c, f\}$ . Above the line  $\bar{\delta}^i$  the first-best transfers can be sustained. For these parameter values of  $\sigma$  and  $\gamma$  the flexible exchange rate regime can sustain some transfers even for discount factors low enough that the common currency regime cannot sustain the first-best,  $\underline{\delta}^f < \bar{\delta}^c$ . However, for different values of  $\sigma$  or  $\gamma$  it is easy to show that  $\underline{\delta}^f > \bar{\delta}^c$  is possible.

### 3.5. Self-enforcing transfers

Self-enforcing transfers will in general be history dependent. However, the results of Thomas and Worrall (1988) show that the long-run distribution is ergodic. In the case with just two states transfers will be history independent as soon as both states have occurred. It easy to find the transfers after both states have been visited. Either the first-best transfer is sustainable or no transfer is sustainable or the transfer will be set so that (13) is binding. This is easy to see as if it weren't binding the transfer could be increased slightly and welfare could be improved.

Suppose that the parameters are such that the first-best transfer is not sustainable but that some self-enforcing transfers are sustainable. To determine the self-enforcing transfer the simply

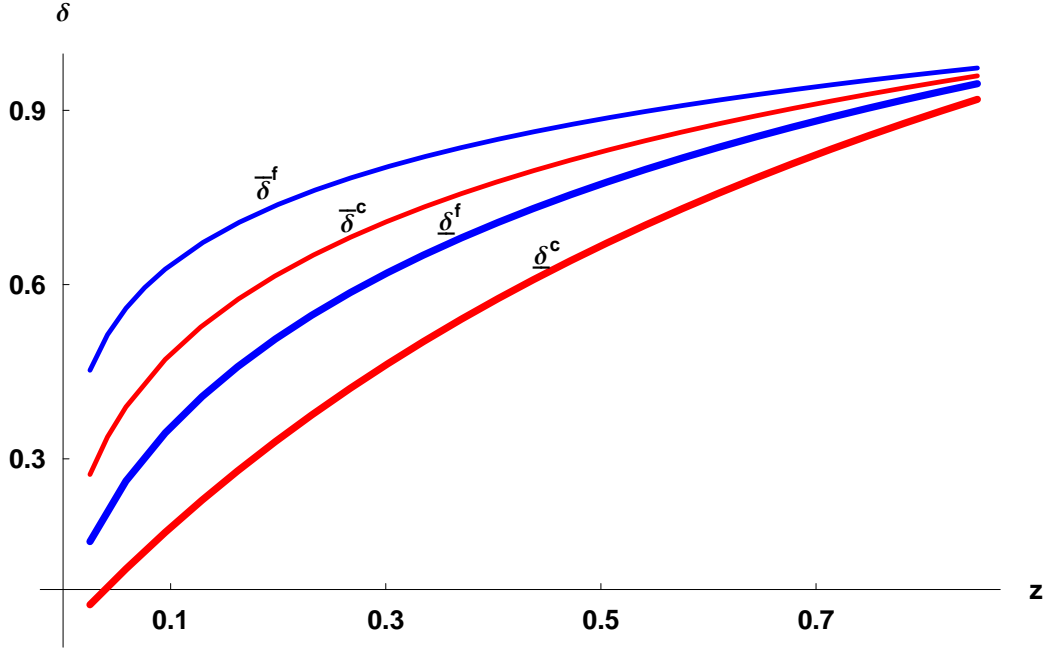


Figure 2: MAXIMUM AND MINIMUM DISCOUNT FACTORS WHICH SUSTAIN NO TRANSFERS OR FIRST-BEST TRANSFERS:  $\sigma = 1.5$ ,  $\gamma = 2$  AND  $\mu = 0.05$ .

involves solving the equation

$$(14) \quad \phi^i(z, \tau^i) \equiv \frac{V(C_B^i(\tau_B^i)) - V(C_B^i(0))}{V(C_G^i(0)) - V(C_G^i(-\tau_G^i))} = \frac{2 - \delta}{\delta}$$

where  $\tau^i = \tau_G^i$  is the transfer made by the country with the good productivity shock and where  $\tau_B^c = \tau_G^c$  and  $\tau_B^f = z^{\frac{\sigma-1}{\sigma}} \tau_G^f$ .

PROPOSITION 5: *There is a unique value of  $\tau^i$  satisfying equation (14) and this value is strictly increasing in  $\delta$ .*

PROOF: Consider the case of the currency union. Differentiating  $\phi^c(z, \tau^c)$  with respect to  $\tau^c$  shows that

$$\text{sign} \frac{\partial \phi^c(z, \tau^c)}{\partial \tau^c} = \text{sign} \frac{V'(C_B^c(\tau^c))}{V'(C_G^c(-\tau^c))} - \phi^c(z, \tau^c).$$

By the concavity of  $V$  we have for any  $\tau^c \in (0, \tau_*^c)$ ,

$$\frac{V(C_B^c(\tau_B^c)) - V(C_B^c(0))}{C_B^c(\tau^c) - C_B^c(0)} > V'(C_B^c(\tau^c)) > V'(C_G^c(-\tau^c)) > \frac{V(C_G^c(0)) - V(C_G^c(-\tau_G^c))}{C_G^c(0) - C_G^c(-\tau^c)}.$$

In varying  $\tau^c$ ,  $C_G^c(0) - C_G^c(-\tau^c) = C_B^c(\tau^c) - C_B^c(0)$  and hence

$$\frac{V'(C_B^c(\tau^c))}{V'(C_G^c(-\tau^c))} - \phi^c(z, \tau^c) < 0.$$

Thus  $\phi^c(z, \tau^c)$  is monotonically decreasing in  $\tau^c$  and hence there is a unique value of  $\tau^c$  satisfying equation (14). As the right hand side of equation (14) is strictly decreasing in  $\delta \in (0, 1)$  the transfer that can be sustained is strictly increasing in  $\delta$ . The proof for the flexible exchange rate case is similar.  $\square$

With constant relative risk aversion preferences the function  $\phi^i(z, \tau^i)$  of equation (14) simplifies considerably so that the transfer  $\tau^i$  can be found by solving the involves solving the equations

$$(15) \quad \phi^c(z, \tau^c) = \phi^c(z^{\frac{1}{\sigma}}, \tau^f) = \frac{(2 - \delta)}{\delta}$$

where

$$\phi^c(z, \tau) = \begin{cases} z^{(\sigma-1)(1-\gamma)} \frac{(1+z^{1-\sigma}\rho\tau)^{1-\gamma}-1}{1-(1-\rho\tau)^{1-\gamma}} & \text{if } \gamma > 1 \text{ and } \gamma \neq 1 \\ -\frac{\ln(1+z^{1-\sigma}\rho\tau)}{\ln(1-\rho\tau)} & \text{if } \gamma = 1. \end{cases}$$

With this specification it is clear that since  $z^{1/\sigma} < z$  we have that  $\tau^f < \tau^c$  so that a greater transfer can be sustained under a currency union than under a flexible exchange rate. In addition it is possible to show that  $\phi^i(z, \tau^i)$  is strictly decreasing in  $z$ . Thus a larger shock (smaller value of  $z$ ) will mean that a larger transfer  $\tau^i$  can be sustained. We therefore have the following result.

**PROPOSITION 6:** *The larger the productivity shock the greater will be the transfer that is sustained in either regime. The common currency union will sustain a greater transfer than the flexible exchange rate regime.*

As it is not possible to obtain an explicit solution for  $\tau^i$  in equation (15) it is difficult to give analytical results of the effect of the different regimes on welfare. We know from Propositions 4 and 6 that the common currency regime sustains greater transfers. However, we also know from Propositions 1 and 2 that the flexible exchange rate system will generate greater welfare if either no transfers or the first-best transfers can be sustained. Thus in comparing welfare there are two opposing effects. The common currency area allows for greater transfers and thus greater sharing of risk and greater welfare. However it does not allow the inefficiencies in the labor market to be offset by adjustments in the exchange rate and this tends to lower welfare. However, using equation (15) it is possible to compute the maximum sustainable transfers for any discount factor and hence compute the expected welfare of each country under flexible exchange rates or a common currency.

Computations show that either effect can dominate. Thus for some parameter values a flexible exchange rate regime generates greater welfare and for other parameter values a common

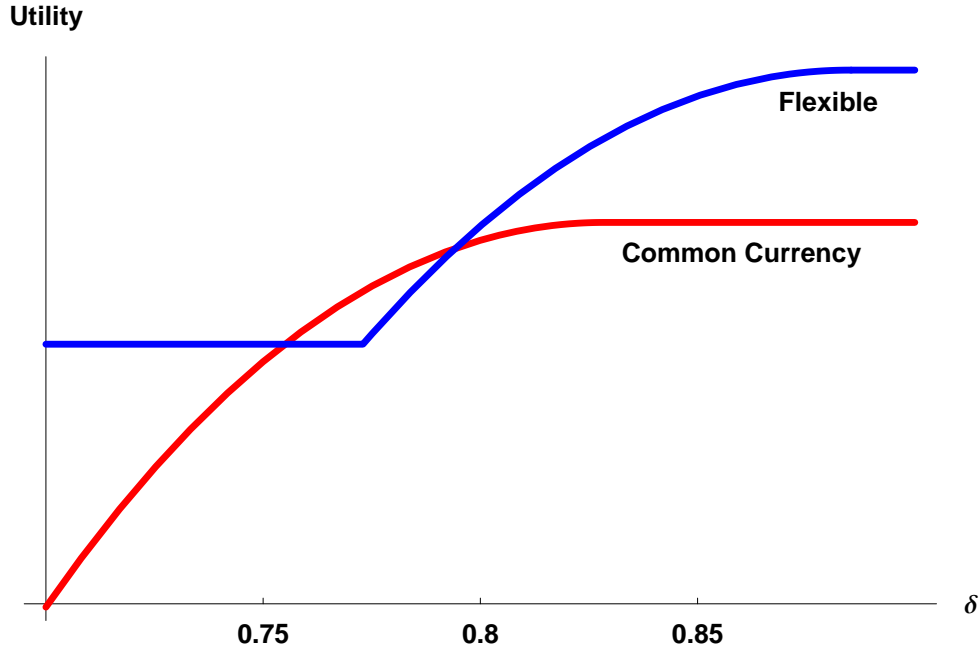


Figure 3: WELFARE COMPARISONS FOR THE COMMON CURRENCY AND FLEXIBLE EXCHANGE RATE REGIMES WITH OPTIMAL SUSTAINABLE TRANSFERS:  $\gamma = 2$ ,  $\sigma = 1.5$ ,  $\mu = 0.05$  AND  $z = 0.5$ .

currency does. As the first-best transfers can be sustained under common currency when less or no transfers can be sustained with flexible exchange rates, there are many parameter values for which higher welfare can be achieved with a common currency than with flexible exchange rates. More interesting perhaps are cases where the common currency cannot sustain first-best transfer, the flexible exchange rate regime can support some transfers but where the common currency yields higher welfare for some parameter values. Such an example is illustrated in Figure 3. The darker line plots the welfare under flexible exchange rates against the discount factor. It is initially horizontal and here no transfers can be sustained but then it rises as some transfers can be sustained and it is again horizontal for higher discount factors when the first-best transfers can be sustained. The lighter line represents expected utility under common currency and it has a similar shape (the lower part where no transfers can be sustained is not illustrated). Following on from Propositions 1 and 2 it is clear that flexible exchange rates dominate for low discount factors where no transfers can be sustained in either regime or for high discount factors where first-best transfers can be sustained in either regime. However, as can be seen the common currency regime does produce higher welfare in an intermediate range of discount factors where neither regime can sustain the first-best transfers.

4. EXTENSIONS

The model can be extended in a number of directions without changing the substantive result that considerations of sustainable assistance are important in determining the relative efficiency of common currency and flexible exchange rate systems. This section outlines some possible extensions.

4.1. *Additional tradable sectors*

In the model outlined both the Home and Foreign countries compete in all goods produced. There may however, be specialized but tradable commodity sectors in each country. Let  $Z_r$  be the output of specialized locally produced goods in country  $r$ . For simplicity consider only trade between  $H$  and  $F$  (set  $\mu = 1$ ) and suppose that the constant elasticity of substitution preferences utility becomes

$$V(C_r) = V \left( \left( \int_0^1 d_{rr}(\zeta)^\rho d\zeta + \int_0^1 m_{sr}(\xi)^\rho d\xi \right)^{\chi/\rho} Z_r^{\frac{1-\chi}{2}} Z_s^{\frac{1-\chi}{2}} \right)$$

where  $(1 - \chi)$  is the share of the specialized sectors (which for simplicity we've assumed is evenly distributed between the two countries). Assuming that the two tradable sectors have constant returns to scale and hire workers who produce at unit productivity, labor demand is  $\ell_r = a_r d_r + Z_r$  and gross domestic product  $Y_r = p_r d_r + Z_r$ . Prices of the competitively produced manufacturing goods is the same as before  $p_r = a_r / \rho$  and the size of the specialized sectors is

$$Z_H = \frac{(1 - \chi)}{2} (Y_H + \varepsilon Y_F) = \varepsilon Z_F.$$

Then one can show that

$$(8') \quad Y_H = \frac{\ell_H}{\rho} \left[ 1 - \frac{(1 - \chi)}{2} \frac{(1 - \rho)}{1 - \chi(1 - \rho)} \left( 1 + \varepsilon \frac{\ell_F}{\ell_H} \right) \right]$$

with a similar expression for  $Y_F$ . Equating demand and supply then gives the following implicit condition for the exchange rate.

$$(10') \quad \left( \frac{a_H}{a_F} \right)^{1-\sigma} = \varepsilon^{1-\sigma} \left[ \frac{(1 - \chi(1 - \rho))\ell_H - \frac{1}{2}(1 - \chi)(\ell_H + \varepsilon\ell_F)}{(1 - \chi(1 - \rho))\varepsilon\ell_F - \frac{1}{2}(1 - \chi)(\ell_H + \varepsilon\ell_F)} \right].$$

One can see that for  $\chi = 1$ , equations (8') and (10') reduce to equations (8) and (10) respectively. In the common currency case  $\varepsilon = 1$  and either  $\ell_H$  or  $\ell_F$  is less than one depending on the technology parameters  $a_H$  and  $a_F$ . In the flexible exchange rate case there is full employment with  $\ell_H = \ell_F = 1$ . As expected experimentation shows that common currency areas is more likely to dominate for a large tradable sector  $\chi$ . Yet the effect of  $\chi$  in the examples we computed is not very significant.

#### 4.2. *Transactions costs*

It is possible to introduce transactions costs into the analysis of the flexible exchange rate regime by allowing for a bid-ask spread. The bid-ask spread can be taken fixed percentage  $\theta$  where

$$\frac{\varepsilon^{bid}}{\varepsilon^{ask}} = 1 - \theta.$$

The introduction of a bid-ask spread has two effects that reduce the attractiveness of the flexible exchange rate regime. First foreign goods become relative more expensive reducing the efficiency gains from trade. Secondly, some of the transfer is lost in transaction and so insurance is also less effective.

#### 4.3. *Alternative stochastic processes*

We have assumed that shocks in the two countries  $H$  and  $F$  are perfectly negatively correlated. This is mainly for convenience and because it is known that under standard assumptions without insurance this is the case in which a flexible exchange rate regime is most dominant. The assumption of perfect negative correlation and symmetry (together with the assumption of constant relative risk aversion) also allowed us to derive some simple analytic formulae for critical discount factors.

It is possible to generalize the stochastic process and allow for any degree of correlation between shocks and also allow for persistence of shocks within countries. With a two state income distribution in each of the two countries, there will be four states for the world economy (with perfect correlation there are just two). This complicates the analysis slightly. With perfect negative correlation either full, partial or no insurance is sustainable. With four income states there will typically be degrees of partial insurance and this needs to be accounted for in the calculations. Nevertheless it is still possible to calculate analytical results. If there are three income states for each country, high medium and low, then there are up to nine states for the world economy and analytic computation becomes difficult. It is possible to compute solutions by numerically by computing and interpolating the appropriate value function. This is not too computationally expensive provided the number of states is relatively small. There is however the further difficulty that with increases in the number of states the time taken to the steady-state becomes longer, so that an appropriate comparison is not with the steady-state but with the long-run expected discounted utility taking into account the transition to the steady-state.

#### 4.4. *Labor market imperfections*

A restrictive assumption of the analysis is that nominal wages are completely inflexible and this stands in contrast to our assumption that the product market is monopolistically competitive. It would be possible to introduce some wage-setting behavior to the model (see for example Danthine and Hunt 1994) to relax this assumption. Introduce wage setting would have two op-

posing effects. First introducing some flexibility into the labor market would make the currency union less unattractive. Secondly it would make the punishment of returning to autarky less severe and hence reduce the amount of insurance that could be sustained. The net effect of reducing the labor market imperfections in the model is therefore ambiguous and something for further analysis.

## 5. CONCLUSIONS

This paper has examined the importance of mutual insurance between countries on the decision to adopt either a common currency or a flexible exchange rate system. Standard analysis suggests that absent any transactions costs the flexible exchange rate system will dominate if either there is no insurance or alternatively if there is full insurance. However, we have shown that if there are restrictions that mean only partial insurance is achievable then this result can be reversed and a common currency can generate higher welfare. The impediment to full insurance we have assumed is that there is no supra-national authority to enforce insurance payments across countries and therefore such insurance must be self-enforcing. The imposition of the constraints of self-enforcement mean that sometimes more insurance can be enforced under a common currency system than a flexible exchange rate system because the threat of no future insurance if there is a breakdown in transfers is more severe in the common currency case.

The paper has presented a simple model to illustrate this possibility. There has been no attempt to calibrate the model as realistic calibration would have to involve a much more sophisticated model of both production and labor market imperfections and a more detailed specification of the stochastic structure of shocks. Rather we decided to illustrate the importance of insurance on the currency union in a model designed to make currency union a priori less desirable. Thus we assumed that there were no transactions costs in the flexible exchange rate system, there was perfect inflexibility in the labor market and shocks were perfectly negatively correlated. Despite these assumptions it has been shown that there are parameter values where a common currency can generate greater welfare. The result should be of interest to policy makers as it shows that any agreement on a system of insurance or fiscal transfers between countries can have an important impact on the monetary decision to adopt a common currency.

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