Limited Commitment Models of the Labour Market

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Aims

- Provide overview of limited commitment risk sharing models of the labour market
- Present a simple general encompassing model and use. arbitrage arguments to characterise solution.
- In particular consider
  - Variable hours.
  - Endoginise the worker's outside option.
- Consider implications for empirical testing.
- Asses available empirical evidence.

Background

- Firms and Workers implicitly agree contingent contracts to share risk.
- They can renege on these contracts when it is to their advantage.
- There are no courts to enforce contracts and low mobility or "lock-in" costs.
- A party called upon to sacrifice current utility to maintain insurance does so in anticipation of future reciprocal benefits.

Motivation

- Standard models of the labour market match data poorly
  - Real wages are weakly correlated with productivity or mildly countercyclical
  - Search models under-predict volatilities of vacancies and unemployment
  - Hours are typically procyclical and negatively correlated with wage rates
- By disassociating wages from productivity and introducing self-enforcing constraints the limited commitment model may explain some of the patterns in labour market data.

Model

- There is an infinite horizon, $t = 1, 2, 3 \ldots \infty$.
- Workers are risk-averse with per-period utility $u(c)$, $u' > 0, u'' < 0$, where $c \geq 0$. Firms are risk-neutral.
- Workers cannot make capital market transactions.
- There is no disutility of work, but hours are fixed so that workers are either employed or unemployed (relaxed later).
- We consider a single match: one worker and one firm.
- There is perfect information within the match.
- Output at time t within the match is $z(s_t) \geq 0$, where s_t is the current state of nature.
- The state of nature s_t follows a time-homogeneous Markov process.
- Workers and firms discount the future with common discount factor $\beta \in (0, 1)$.

Payoffs

- Let $h_t := (s_1, s_2, \ldots, s_t)$ be the history at t.
- A contract specifies $(w_t(h_t))_{t=1}^{T} = ((w_1(s_1), w_2(s_1, s_2), w_3(s_1, s_2, s_3), \ldots))$, where $w_t(h_t) \geq 0$ is the wage at t after history h_t, and $T > 1$ is the (random) date at which the contract is terminated.
- The value (discounted utility) of the outside option for the worker and firm respectively is denoted by $\chi_w(s)$ and $\chi_f(s)$ in state s.
- If there is a termination (agreed or otherwise) the firm and worker both incur costs C_f and C_w.

Timing

- Both agents observe the current state of nature, s_t at the start of each period.
- At this point either party can quit and take their outside option.
- Otherwise, they trade at the agreed terms, in which case the value of output $z(s_t)$ is realised, and the firm makes the wage payment according to the contract.

Continuation utilities are:

$$V_t(h_t) := u(w_t(h_t)) + E\left[\sum_{t'=t+1}^{T-1} \beta^{t'-t} u(w_{t'}(h_{t'})) + \beta^{T-t}\chi_w(s_T) \mid h_t\right],$$

$$\Pi_t(h_t) := z(s_t) - w_t(h_t) + E\left[\sum_{t'=t+1}^{T-1} \beta^{t'-t}(z(s_{t'}) - w_{t'}(h_{t'})) + \beta^{T-t}\chi_f(s_T) \mid h_t\right].$$

The contract is said to be self-enforcing if the following hold for all dates t, $T-1 \geq t \geq 1$, and for all positive probability h_t:

$$V_t(h_t) \geq \chi_w(s_t) - C_w, \tag{1}$$

$$\Pi_t(h_t) \geq \chi_f(s_t) - C_f. \tag{2}$$

Constrained Efficient Contracts

Efficient contracts are thus solutions to the following problem:

$$\max_{(w_t(h_t))_{t=1}^{T}} \Pi_1\left(h_1\right)$$

subject to the self-enforcing constraints (1), (2), and

$$V_1\left(h_1\right) \geq \bar{V}_1 .$$

The term $\bar{V}_1$ measures how much utility the worker gets from the relationship, and as this is varied across feasible values (i.e. values for which self-enforcing contract exist), all efficient contracts are traced out.

Lemma 1

Lemma

In an efficient contract in which the firm's (worker's) participation constraint is slack at $t+1$, wages cannot fall (rise) between t and $t+1$.

Proof.

See diagram. □

Proof of Lemma 1

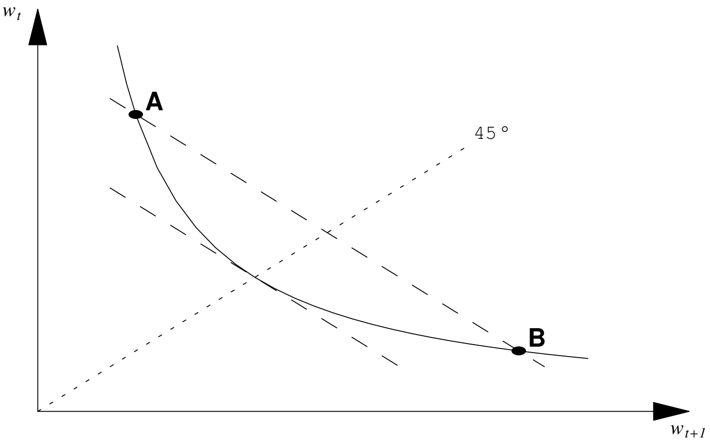

Figure: Illustration of Proof of Lemma 1

Binding participation constraint

- Let $\left(w_t(\bar{V}_1; s)\right)_{t=1}^{T}$ be a constrained efficient contract starting from state $s_1 = s$.
- Define $\underline{w}_s := w_1(\chi_w(s) - C_w; s)$.
- Note:
  i. $\underline{w}_s$ is unique.
  ii. It is optimal at any date t in state s to set $w_t = \underline{w}_s$ whenever $V_t(h_t) = \chi_w(s) - C_w$.
- Likewise define $\bar{w}_s$ to be the period 1 optimum wage starting in state s which delivers profits of exactly $\chi_f(s) - C_f$.

Lemma 2

Lemma

If $V' > V$, then $w_1(V'; s) > w_1(V; s)$.

Proof.

Assume otherwise that $w_1(V'; s) \leq w_1(V; s)$. Then there exists some future path h_{t+1} with $w_{t+1}(V'; s) > w_{t+1}(V; s)$ for the first time. This implies that wage growth between t and $t+1$ is greater in the V' case.

Then from Lemma 1 either: (i) the worker's participation constraint binds at $t+1$ for the V' contract; or (ii) the firm's constraint binds in the V contract; or both.

In case (i) wages are weakly lower in the V' contract than in the V contract until $t+1$ and since the participation constraint binds the V contract cannot offer less; contrary to assumption. Likewise in case (ii) there is a contradiction. □

The updating rule

Proposition

An optimal contract evolves according to the following updating rule. In state $s \in S$ either (a) the contract (always) terminates, or (b) there is associated a minimum and a maximum wage, $\underline{w}_s$ and $\bar{w}_s$ respectively ($\underline{w}_s \leq \bar{w}_s$), such that in an optimal contract if at date $t+1$ state s_{t+1} occurs then w_{t+1} is updated from w_t by

$$w_{t+1} = \begin{cases} \bar{w}_{s_{t+1}} & \text{if } w_t > \bar{w}_{s_{t+1}}, \\ w_t & \text{if } w_t \in \left[\underline{w}_{s_{t+1}}, \bar{w}_{s_{t+1}}\right], \\ \underline{w}_{s_{t+1}} & \text{if } w_t < \underline{w}_{s_{t+1}}. \end{cases}$$

Proof of updating rule

Proof.

(i) Suppose that $w_t \in \left(\underline{w}_{s_{t+1}}, \bar{w}_{s_{t+1}}\right)$. If the worker's participation constraint at $t+1$ in state s_{t+1} binds, $w_{t+1} = \underline{w}_{s_{t+1}}$, wages fall and the firm's constraint is slack $(w_{t+1} \neq \bar{w}_{s_{t+1}})$. But this contradicts Lemma 1 that wages do not fall. Thus the worker's constraint does not hold and from Lemma 1 wages cannot rise. Likewise as $w_t < \bar{w}_{s_{t+1}}$ the worker's constraint cannot bind, and wages cannot fall. Thus wages remain constant.

(ii) Suppose $w_t \leq \underline{w}_{s_{t+1}}$. If the worker's constraint does not hold $(V_{t+1} > \chi_w(s_{t+1}) - C_w)$, by Lemma 1 wages cannot rise, so $w_{t+1} \leq \underline{w}_{s_{t+1}}$. However, $V_{t+1} > \chi_w(s_{t+1}) - C_w$ would imply by Lemma 2 that $w_{t+1} > \underline{w}_{s_{t+1}}$, a contradiction. So the constraint binds and $w_{t+1} = \underline{w}_{s_{t+1}}$.

(iii) A symmetrical argument establishes that $w_{t+1} = \bar{w}_{s_{t+1}}$ if $w_t > \bar{w}_{s_{t+1}}$. $\square$

Variable Hours

- Contracts specifies profile for hours worked in addition to a profile for wages.
- Hours are chosen efficiently

$$-\frac{u_H(c_t(h_t), H_t(h_t))}{u_c(c_t(h_t), H_t(h_t))} = z(s_t).$$

- The modified updating rule is

$$\lambda_{t+1} = \begin{cases} \bar{\lambda}_{s_{t+1}} & \text{if } \lambda_t > \bar{\lambda}_{s_{t+1}}, \\ \lambda_t & \text{if } \lambda_t \in \left[\underline{\lambda}_{s_{t+1}}, \bar{\lambda}_{s_{t+1}}\right], \\ \underline{\lambda}_{s_{t+1}} & \text{if } \lambda_t < \underline{\lambda}_{s_{t+1}} \end{cases}$$

where

$$\lambda_t(h_t) = u_c(c_t(h_t), H_t(h_t)).$$

Frisch-type demand functions

- The Frisch-type demand functions $c(\lambda, z)$ and $H(\lambda, z)$ satisfy

$$u_c(c, H) = \lambda$$
$$-u_H(c, H) = \lambda z.$$

- Provided leisure is a normal good, the hours function $H(\lambda, z)$ is increasing in λ and z.
- $c(\lambda, z)$ is decreasing in λ provided consumption is normal.
- In the limited commitment contractual solution, consumption and hours satisfy these two equations where consumption equals earnings, $c_t(h_t) = w_t(h_t)H_t(h_t)$ and $\lambda_t(h_t)$ satisfies the updating rule.
- In the standard intertemporal model of labour supply the above equations apply with $z = w$ and with λ determined by an Euler equation of the form $\lambda_t = (1 + r_t)\beta \mathrm{E}[\lambda_{t+1}]$.

Ex ante wage payments

- The contract space may be extended by considering an up-front payment at date t depending on the history h_{t-1} (Gauthier-Poitevin-Gonzalez, 1997).
- If the agent who is likely to be constrained makes an up-front payment this can relax the (ex post) participation constraints.
- This introduces ex ante participation constraints on the up-front payments.
- The effect is to make the updating rule history dependent in general.

Quitting costs

- So far we assumed the termination costs C_f and C_w are incurred by both parties whenever either party terminates the relationship.
- Suppose instead there are direct costs to reneging (psychic, legal etc.), p_f and p_w.
- Then termination may be history dependent.

Efficient termination

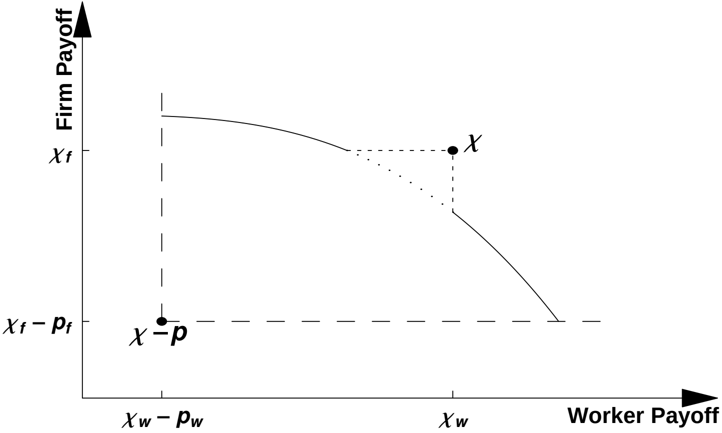

Figure: The Pareto-Frontier, Surplus and Actions

Implications for wage rates

- Outside options have implications for wages: Current wages responsive to best and worst outside option (as measured by say, unemployment rate) since the start of the employment relationship.
- Cohort effects tend to disappear over time (as participation constraints become binding.
- Conditions prior to employment have no effect.
- There will be an asymmetric response to changes in outside options.
  - If the worker is close to the participation constraint, then an increase in unemployment may have no effect on wages, but a decrease in unemployment may increase wages.
  - A large increase in unemployment may cause the firm's constraint to bind and wages to fall.
- Worker heterogeneity (outside options) can matter.

Implications for hours

- If λ is fixed, then looking at different cohorts of workers hours and wage rates are negatively correlated.
- Cohort effects disappear so the cross-sectional variation in wages and hours across employees should be lower with increasing tenure.
- There will be an ambiguous or weak effect of changes in productivity on hours and wage rates.
- The growth rates in wages and hours will vary across workers of different tenure.

Search frictions

- Two papers by Sigouin (2004) and Rudanko (2006).
- Outside option is contract offered by other firms.
- Probability of new match when quitting but prob ability of unemployment.
- Probability that match dissolves.
- Rudanko has Cobb-Douglas matching function depending on number of vacancies and number of unemployed.
- Outside option of firm is zero by competition.

One-sided commitment

- Firms can commit to contracts.
- Workers cannot but are perfectly mobile and can find new contracts immediately (Beaudry-DiNardo, 1991).
- By Lemma 1 wages cannot fall.
- By competition firm profit driven to zero.
- Outside option $\chi_w(s)$ is the utility from efficient contract given zero profits.
- Workers receive wage below productivity initially.
- Current wage determined by tightest labour market conditions during tenure.
- Contrast with current unemployment level (spot market) and unemployment level at time of hiring (full commitment).

Empirical evidence I

- Macis (2006) uses Italian data.
  - Finds best and worst unemployment rates significant in explaining wages.
  - Evidence of disappearance of cohort effects.
  - Some asymmetry in response of wages but wages are still more downward sticky than the model predicts.
  - Prior conditions do no matter, worker heterogeneity does.
- Beaudry & DiNardo (1995) run regression of the form

$$\Delta \ln H_{j,\tau+t} = \alpha_1 \Delta \ln w_{j,\tau+t} + \alpha_2 \Delta \ln z_{j,k,\tau+t} + \alpha_3 \Delta X_{j,\tau+t} + \epsilon_{j,\tau+t}.$$

  - They find α_1 statistically negative.
  - A 1% reduction in unemployment leads to a 3-4% increase in wage rate and $\frac{1}{2}$-1% reduction in hours for a given productivity

Empirical Evidence II

- Rudanko uses US data to analyse volatilities in wage and vacancy/unemployment rates.
  - With commitment wages vary too little with productivity.
  - To match the two-sided model if the replacement ratio is very high.
- Testing the one-sided commitment model.
  - Beaudry-DiNardo (1991) find strong evidence in favour of the model.
  - Several studies broadly support this conclusion (McDonald & Worswick, 1999; Grant, 2003; Shin & Shin 2007; Devereux & Hart 2007).

Conclusions

- Presented an overview of self-enforcing labour contracts under risk sharing
- Presented firm-worker model general enough to encompass that considered in the literature.
- Used variational arguments rather than dynamic programming.
- Considered how worker's outside option can be endogenised.
- Concluded that limited commitment does does broadly help to explain some empirical regularities.

Areas for further work

- Can test properly discriminate against alternatives? (capital market imperfections, hold-up)
- Can the model explain firm level data (e.g. large firms pay higher wages, fast-growing firms pay lower wages etc.).

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