Self-Enforcing Wage Contracts Redux

by

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This paper provides a personal perspective on self-enforcing wage contracts. We present a simple version of the model of Thomas and Worrall (1988) and explains its motivation, contribution and methodology. We discuss some of its limitations, the development of literature and its connection to the literature on relational contracting with an observable effort cost. We suggest some open questions for the future development of the literature.

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1 Introduction

This paper provides a personal perspective on self-enforcing wage contracts. We present a simple version of the model of Thomas and Worrall (1988) and explain its motivation, contribution and methodology. We discuss some of its limitations, the development of literature and its connection to the literature on relational contracting with an observable effort cost as in MacLeod and Malcomson (1989). We suggest some open questions for the future development of the literature.

2 Background

2.1 Motivation

One of the main aims of labour economics is to understand why wages are comparatively stable. One strand of the literature, developed in the 1970s, often called implicit contracts (see, for example, Baily, 1974; Azariadis, 1975), argued that employers would insure workers against fluctuations in marginal productivity thereby stabilising the real wage. At its most simplistic, suppose the wage w is a random variable that reflects changes in marginal productivity.

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Before the realisation of the wage, the worker has an expected utility of $\mathbb{E}[u(\tilde{\mathrm{w}})]$ and the expected cost to a risk-neutral employer is $\mathbb{E}[\tilde{\mathrm{w}}]$. Provided the worker is risk averse, then there is an ω such that $u(\omega) > \mathbb{E}[u(\tilde{\mathrm{w}})]$ and $\omega < \mathbb{E}[\tilde{\mathrm{w}}]$. An implicit contract that offers a fixed wage of ω improves the expected utility of the employee and reduces the costs to the employer. The theory starkly predicts real wage stability.

It is hard to imagine now that this theory was novel and innovative at the time.[1] discussion of these issues. There are however, two obvious and important deficiencies. First, one might suppose that the marginal product is not perfectly observable. For example, the marginal product might depend randomly on the effort of the worker. This gives rise to the classic moral hazard problem. The wage will not be perfectly stabilised because there is a trade-off between incentives and risk sharing (see, for example, Holmström, 1979; Grossman and Hart, 1983). Second, suppose that there are no issues of observability, for example, that the spot wage perfectly reflects marginal productivity, but that the implicit contract does not commit the firm to the worker nor the worker to the firm. In this case, if the realised state has $\mathrm{w}(s) > \omega$, then the worker will be tempted to renege and work at the higher spot market wage. If, on the other hand, $\mathrm{w}(s) < \omega$, the firm will be tempted to renege and hire a new worker at the lower spot market wage. In a static setting, the implicit contract unravels and only the equilibrium wage is the spot market wage. If however, the firm and the worker have a long-term relationship, the short-term incentive to renege is offset by the longer-term benefit of having a more stable wage in the future. There is a trade-off between risk-sharing and commitment. In some cases this trade-off is trivial. If the long-term relationship lasts for a fixed and known length, then in the last period, the implicit contract unravels just like in the static case, and hence, the penultimate period is just like the last period and so there is backward unravelling as is the case in many finitely repeated games. Equally, if the firm and worker have a low discount factor, the future benefits cannot offset the short-term benefits of reneging, or if both the firm and worker have a sufficiently high discount factor, then it will be possible to sustain the fixed wage implicit contract. The aim of Thomas and Worrall (1988) was to study

[1]The idea that the firm provides insurance to the worker is, of course, not self-evident. If financial markets were complete, it would not be necessary for the firm to provide insurance. However, financial markets are not complete. In particular, workers cannot issue and sell state-contingent claims to their labour income. The informational and enforcement issues of providing insurance may be best solved within the firm. For example, if insurance is offered by a third party, both the firm and the worker would have an incentive to report bad states. The assumption that the firm is less risk averse than the worker was influenced by the Knightian view that entrepreneurs and workers self-select, with the more risk averse becoming workers. Equally, firms may be less risk averse if idiosyncratic firm risk can be diversified away by stock holders. Furthermore, the empirical evidence suggested that labour income fluctuates less than firm revenue suggesting that there is insurance of workers within firms. Guiso, Pistaferri, and Schivardi (2005) and subsequent literature find that workers are well insulated from transitory components of diversifiable firm risk, although less so against permanent shocks to the firm. See Pagano (2020) for a comprehensive

this trade-off between risk-sharing and commitment in an infinitely repeated model when the firm and worker discount is neither too high nor too low.

2.2 History of the Paper

The paper Thomas and Worrall (1988) was conceived when both authors were research officers of the Social Science Research Council project on Risk, Information and Quantity Signals in Economics, which was led by Frank H. Hahn and ran from 1978 to 1994 at Cambridge University's Department of Applied Economics.[2] The paper was written in late 1983 and early 1984 and an initial working paper version was disseminated as the University of Cambridge, Economic Theory Discussion Paper No. 74, April 1984. The paper was first presented at the project's famous "Quaker Meetings" held in Frank Hahn's Office.[3] Among those present at this first presentation of the paper were Margaret Bray, Partha Dasgupta, Tim Kehoe, David Newbery and Robert Solow. The first external presentation of the paper was at a session of the Econometric Society European Meetings held in Madrid on 7th September 1984 that was chaired by Mordecai Kurz.

The paper was initially submitted to *Econometrica* in Autumn 1984. The paper was later rejected and the referees raised three main concerns. First, that the underlying game between the firm and worker was not fully specified. Second, that the results would depend on the nature of bargaining between firm and worker and third, that the model is partial rather than general equilibrium. Although rather late now, we will discuss some of these concerns below and how they have been addressed in the literature, in particular in the issue of endogenous separation when the outside option is another contract instead of the spot wage. A revised version of the paper was subsequently submitted to the *Review of Economic Studies* in February 1986 and conditionally accepted on 12th January 1988.[4] Although the paper has been cited, citations were initially sluggish and the first time the Google Scholar citation count past double figures in a given year was in 2000, twelve years after publication.

2.3 Influences

We were not the first to think about the trade-off between risk-sharing and commitment in the context of wage contracts. Harris and Holmström (1982) addressed a similar issue but had assumed that information is symmetric and incomplete and that firms could commit whereas workers could not. Bester (1984)

[2]Typically, one junior research officer was appointed each year for a period of two years. Prior junior research officers on the project were Louis Makowski, Mark Machina, and Ben Lockwood. Those that followed included: David Kelsey, Paul Seabright, Chris Doyle, Luca Anderlini, and Piero Gottardi.

[3]The idea of the Quaker meetings was that anyone who was moved by the spirit would talk about their current research. Of course, there was some pre-meeting co-ordination between some members of the project as to whom the spirit was going to move on a particular day.

[4]The managing editor was John H. Moore. The acceptance letter included a list of 27 points to address.

analysed the case where neither firm nor worker could commit but assumed that the wage contract could only depend on the current state, not the whole history of states. Thomas and Worrall (1988) allowed for wages to be history-dependent and assumed that neither firm nor worker could commit. As we show below, a model where one side can commit and the model of Bester (1984) are particular special cases of the more general model.[5]

At the time we were writing the paper there was a great deal of interest in studying repeated games with discounting and an early version of Dilip Abreu's paper on infinitely repeated games with discounting (Abreu, 1988) was circulating. At around the same time, ideas in that paper were being refined and developed with David Pearce and Ennio Stacchetti to show how the sub-game perfect equilibria of infinitely repeated games had a recursive structure: the outcome profile of the continuation game must itself be an equilibrium profile and players choose actions to maximize the sum of the current payoffs and the discounted continuation payoffs, which were later published in Abreu, Pearce, and Stacchetti (1986). In addition, during the Michaelmas term of 1983, we both attended a lecture course on dynamic programming given by Peter Whittle at Cambridge University. Therefore, we knew that our problem of finding an optimal contract could be formulated as a dynamic programming problem. The way this could be done was to use the *promised utility* as a state variable. We were not alone in recognising the usefulness of this approach. Spear and Srivastava (1987) independently developed a similar approach to study a repeated moral hazard problem.[6] Concurrently, we were working on using the same approach to study a repeated adverse selection problem (Thomas and Worrall, 1990) and independently Green (1987) had used recursive techniques to solve a similar problem.[7]

The title of our paper was motivated by the interpretation of a Nash equilibrium as a self-enforcing agreement from which no player has an incentive to deviate. Telser (1980) had used the term self-enforcing agreement to apply to any situation where each party decides unilaterally whether to continue or stop with his or her relationship with the other parties depending on whether the current gain from stopping is inferior to, or exceeds, the expected present value of the gains from continuing. Thus, the title of our paper was an amalgam of self-enforcing agreement and implicit wage contracts. As the terminology has developed in the subsequent literature, if we were writing it today, we might have called it "Wage contracts with limited commitment" or "Wages with unenforceable contracts".[8]

[5] We became aware of Bester (1984) when writing our paper. We were aware of Holmström (1983) which considered a finite horizon model where the worker is unable to commit. However, we appreciated the relevance of Harris and Holmström (1982) only after we had written the initial draft of the paper.

[6] By coincidence, both Thomas and Worrall (1988) and Spear and Srivastava (1987) were submitted to the *Review of Economic Studies* in February, 1986.

[7] An alternative dual approach to recursive contracting that has some computational advantages is provided by Pavoni, Sleet, and Messner (2018) and Marcet and Marimon (2019).

[8] Another important influence was Grout (1984), which examined a hold-up problem where the wage was determined by a Nash bargain after an investment had been made.

3 Model

There is an infinite sequence of dates, $t = 1, 2, ..., \infty$ and a finite set of states $s_t \in \{1, 2, ..., S\}$, $S \geq 2$, at each date. There are two types of agents, firms and workers. Firms employ one worker at each date. A worker supplies one indivisible unit of labour which the firm converts into one revenue unit of output. There are no costs or delay to a worker in moving from one firm to another and no cost to a firm in hiring or firing workers: specifically, after the spot market wage is observed either agent might move to the spot market immediately.[9] Each agent can trade labour on the spot market or negotiate a contract before trade takes place, at date $t = 0$, which specifies the wage payment at every date-event pair. Each agent has perfect foresight; they know the spot market wage at every date-event pair, and each is a perfect competitor; they take the spot market wage as given. No agent can pre-commit to a wage contract, so they will renege if it is to their advantage. Therefore, the only feasible contracts are the self-enforcing ones in which neither firm nor worker ever has an incentive to renege.

The following assumption is made about the effect of breach of contract upon an agent's future utility.

Assumption 1 *An agent who reneges on a contract must from then on trade on the spot market forever.*

Assumption 1 is a useful and commonly-used benchmark. It is a strong assumption but can be relaxed. For example, one may consider that in the event of a breakdown, the firm and worker suffer an additional state-contingent loss of expected discounted utility (see, for example, Ligon, Thomas, and Worrall, 2002) or that agents trade on the spot market for a fixed number of periods. Such relaxations will not affect the qualitative properties of the characterisation we give below. We will also show in Section 5.1 that our solution remains unchanged if Assumption 1 is replaced by a requirement that the solution be renegotiation-proof. Furthermore, in Section 5.3 we show how the solution is changed when instead it is assumed that a breach by the worker might lead to a contract with a new firm.

All workers are infinitely lived, risk averse and have an identical per period, state independent utility function, $u = u(\omega)$, where ω is the wage paid.

Assumption 2 $u(\omega)\colon [a, b] \to \mathbb{R}$ *is differentiable and strictly increasing and strictly concave.*

All firms are identical, infinitely lived and risk neutral. Both firms and workers discount the future by a common factor $\beta \in (0, 1)$.

Each state is identified solely by the spot market wage $\mathrm{w}(s)$. As a convention $\mathrm{w}(s) > \mathrm{w}(s-1)$. The stochastic process generating spot market wages is assumed to satisfy:

[9] We consider the implication of relaxing these assumptions in Section 5.3.

Assumption 3 *States are identically and independently distributed such that* $a < \mathrm{w}(1) < \mathrm{w}(S) < b \leq 1$. *The probability of state s is* p_s *where* $\sum_{s=1}^{S} p_s = 1$.

Assumption 3 ensures utility is defined for every possible spot market wage and, since one unit of labour produces one revenue unit of output and $b \leq 1$, firms have an incentive to hire labour in every state. We define $\mathrm{w}^* := \sum_{s=1}^{S} p_s \mathrm{w}(s)$ as the expected spot wage and $\mathrm{w}_* := u^{-1}(\sum_{s=1}^{S} p_s u(\mathrm{w}(s)))$ as the certainty equivalent spot wage. By the concavity of the utility function, $\mathrm{w}^* > \mathrm{w}_*$. It is possible to generalise to the case where states follow a Markov process instead of assuming they are i.i.d. All the results below apply to the case of a Markov process and the difference is quantitative rather than qualitative. We make the i.i.d. assumption because it simplifies notation and it emphasizes that the history dependence in wages that we derive does not depend on any persistence in the spot market wage.

Let $h_t := (s_1, s_2, ..., s_t)$ be the history of states at t. We start in some given state s_1 at date $t = 1$ and hence, $h_1 = s_1$

A contract, δ, is an infinite sequence $(\omega(h_t))_{t=1}^{\infty}$, where $\omega(h_t) \geq 0$ is the wage paid at date t after history h_t.

For any contract, δ, and any history, h_t, the *net* gain to the worker, including the benefit or loss at date t and the long-run expected future benefit, is

$$U_t\left(h_t;\delta\right) := u(\omega(h_t)) - u(\mathrm{w}(s_t)) + \mathbb{E}\left[\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\left(u(\omega(h_\tau)) - u(\mathrm{w}(s_\tau))\right) \middle| \, h_t\right],$$

where $\mathbb{E}$ denotes expectation. If $u(\omega(h_t)) < u(\mathrm{w}(s_t))$, then the worker will have a short-term incentive to renege and work at the spot market wage, which must be counterbalanced by the long-term benefits from compliance. Likewise, the firm's *net* gain, including the benefit or loss at date t and the long-run expected future benefit, is

$$\Pi_t\left(h_t;\delta\right) := \mathrm{w}(s_t) - \omega(h_t) + \mathbb{E}\left[\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\left(\mathrm{w}(s_\tau) - \omega(h_\tau)\right) \middle| \, h_t\right].$$

If $\omega(h_t) > \mathrm{w}(s_t)$, then the firm will have a short-term incentive to renege and hire at the spot market wage.

Definition 1 *The contract is said to be* self-enforcing *if the following hold for all histories* h_t*:*

$$U_t\left(h_t;\delta\right) \geq 0, \tag{1}$$

$$\Pi_t\left(h_t;\delta\right) \geq 0. \tag{2}$$

Inequality (1) is the worker's *participation constraint* that says that at any point in the future the contract must offer at least what a worker can get by quitting to work at spot wages, while (2) is the corresponding constraint for the firm.

Let $\Lambda(s_t)$ be the set of contracts which satisfy the self-enforcing constraints after the history h_t. Since all the self-enforcing constraints are forward looking and the time horizon is infinite, $\Lambda(s_t)$ depends only on the current state s at time t, but is independent of the actual history h_{t-1}.

Definition 2 *Let $\Lambda(s_t)$ be the set of contracts which satisfy the self-enforcing constraints* (1) *and* (2) *after the history (h_{t-1}, s_t).*

We are interested in *constrained efficient contracts*, that is to say contracts which are self enforcing and are not Pareto dominated by any other self-enforcing contracts.

Definition 3 *An efficient contract δ is the solution to the following problem:*

Problem A
$$\sup_{\delta \in \Lambda(s_1)} \left\{ \Pi_1 (s_1; \delta) \mid U_1 (s_1; \delta) \geq \widehat{U}_1 \right\}.$$

The term $\widehat{U}_1$ measures how much utility the worker gets from the relationship starting in the given state s_1 at the initial date. As $\widehat{U}_1$ is varied across feasible values (i.e. values for which a self-enforcing contract exists), the constrained Pareto frontier is traced out.

4 Results

4.1 Interval Characterisation

Despite the apparent complexity of the problem which allows for any general history dependence, the (constrained) efficient contract is characterised in a simple and intuitive way.

Proposition 1 *An efficient contract has an interval of wages $[\underline{\omega}_s, \bar{\omega}_s]$ for each state s. For any history (h_t, s), the contract wage at date $t+1$ satisfies*

$$\omega(h_t, s) = \begin{cases} \bar{\omega}_s & \text{for } \omega(h_t) > \bar{\omega}_s, \\ \omega(h_t) & \text{for } \omega(h_t) \in [\underline{\omega}_s, \bar{\omega}_s], \\ \underline{\omega}_s & \text{for } \omega(h_t) < \underline{\omega}_s. \end{cases}$$

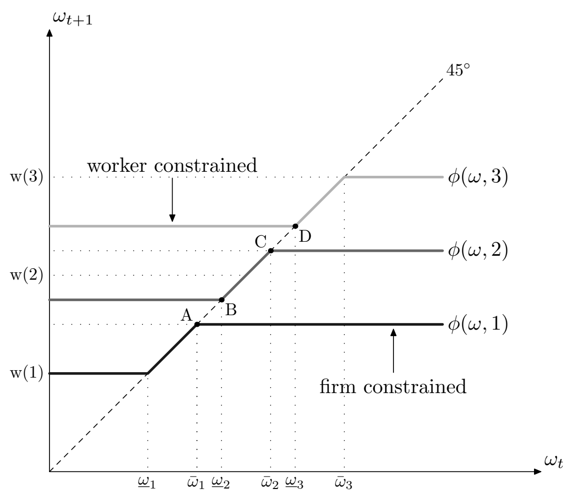
For any two states $k > s$, $\bar{\omega}_k > \bar{\omega}_s$ and $\underline{\omega}_k > \underline{\omega}_s$. Additionally, $\mathrm{w}(s) \in [\underline{\omega}_s, \bar{\omega}_s]$ *for all $s \in \{1, 2, ..., S\}$ with $\underline{\omega}_1 = \mathrm{w}(1)$ and $\bar{\omega}_S = \mathrm{w}(S)$.*

Consider a particular state s and the associated interval $[\underline{\omega}_s, \bar{\omega}_s]$. The proposition says the following. If the wage in the previous period is in the interval of the current state, $\omega(h_t) \in [\underline{\omega}_s, \bar{\omega}_s]$, then the current wage, $\omega(h_t, s)$, is unchanged from the previous period, that is, $\omega(h_t, s) = \omega(h_t)$. Otherwise, the current wage is set at the top of the interval if $\omega(h_t) > \bar{\omega}_s$, or at the bottom of the interval if $\omega(h_t) < \underline{\omega}(h_t)$. This gives a simple and intuitive rule: given the current state, keep the wage constant or adjust it to the smallest possible extent to bring it within the relevant interval. Since $\underline{\omega}_s$ and $\bar{\omega}_s$ correspond to giving the worker and firm a zero net gain starting from state s, wages adjust but only when a constraint of either the firm or worker is binding.

While the interval end-points, $\underline{\omega}_s$ and $\bar{\omega}_s$ correspond to giving the worker and firm a zero net gain starting from state s, it should be stressed that they are not the least and greatest wages that could feasibly be paid subject to the self-enforcing constraints being satisfied, but are the least and greatest wages it is *optimal* to pay. This will become clearer when we study the solution below.

Proposition 1 describes a *stochastic recursive sequence* for the wage dynamics. The wage at time $t+1$ depends on the wage at time t and the state at time $t+1$. That is, $\omega_{t+1} = \phi(\omega_t, s_{t+1})$. Each function $\phi(\omega, s)$ is _/‾-shaped with the upward sloping part along the 45° line. The lower arm of the _/‾-shape is where the worker is constrained: any lower wage would mean the worker would prefer to leave the firm and take the higher spot wage. The upper arm of the _/‾-shape is where the firm is constrained: any higher wage would mean the firm would prefer to hire a new worker at the lower spot wage. This is illustrated in Figure 1 for a case with three states where none of the three intervals overlap.

Figure 1
Example with 3 states where the intervals do not overlap.



The four key properties of the wage dynamics following from Proposition 1 can be illustrated by reference to Figure 1:

History Dependence Wages are dependent on the history of states. For example, in state 2 in Figure 1, the wage will be at point B with a wage of $\underline{\omega}_2$ if the state in the previous period was state 1, whereas it will be at point C with a wage of $\bar{\omega}_2$ if the state in the previous period was state 3. The history matters for the current wage.

Amnesia The history dependence is limited. Suppose that the wage is at point C in Figure 1 with a wage of $\bar{\omega}_2$, then the previous history of how this point is reached, whether there was a short or long sequence of state 3s in the past, is irrelevant. All that matters is that the firm is constrained at that point in state 2. In this sense, there is *amnesia* and the previous history is forgotten once a constraint is hit.

Convergence Although the wage ω can be at any point within the interval $[\underline{\omega}_s, \bar{\omega}_s]$, it can be shown that there is convergence to a long-run distribution with its support on a finite set. That is, there is a finite ergodic set of wages.[10] For example, it can be seen from Figure 1 that the long-run distribution has only four possible wages given by the four points, A, B, C, D in the diagram.

Back-loading Suppose that initially the surplus is extracted by the firm so that the initial expected discounted utility of the worker is zero. In this case, the initial wage is set to the lower endpoint of the relevant interval. For example, in Figure 1, the initial wage would be at A, B, or D. Viewing this as the initial wage distribution, it is stochastically dominated by the long-run wage distribution at A, B, C, D. In fact, it can be shown that the wage distribution at date $t+1$ weakly stochastically dominates the wage distribution at date t. In this sense, wages are *back-loaded* into the future, a result familiar from the two-period model of Holmström (1983). Conversely, if the worker can extract all the initial surplus, then wages are *front-loaded*.

If the firm can commit, then the firm's constraint is irrelevant and each function $\phi(\omega, s)$ will be $\_\diagup$-shaped with no upper arm. In the case of Bester (1984) where the contract wage can depend only on the current state, each function $\phi(\omega, s)$ is a horizontal line independent of the previous wage ω. The long-run distribution can have at most the same number of wages as there are states.

4.2 Finding the endpoints

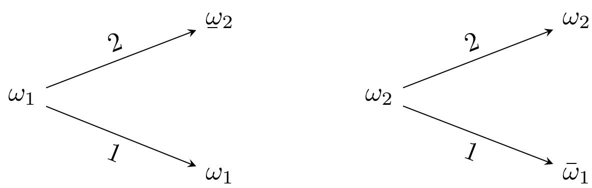
Proposition 1 shows how the wage is updated but doesn't determine the intervals $[\underline{\omega}_s, \bar{\omega}_s]$. In the next subsection we show how the intervals are determined using a dynamic programming methodology. However, we first show how the interval endpoints can be determined from knowledge of Proposition 1 without the need to compute the value functions explicitly. We do this in a simple example with two states w(2) > w(1). We know from Proposition 1 that $\underline{\omega}_1 = \mathrm{w}(1)$ and $\bar{\omega}_2 = \mathrm{w}(2)$. Therefore, there are two cases to consider depending on whether $\underline{\omega}_2 \gtreqless \bar{\omega}_1$.

Let $\Pi_s(\omega)$ be the net expected discounted profit of the firm starting from a wage ω in state s and let $U_s(\omega)$ be the net expected discounted utility for the

[10] A set E of wages is ergodic if once in the set, there is no probability of moving outside of the set and there is no proper subset with this property.

worker starting from a wage ω in state s. Since the firm is constrained at a wage of $\bar{\omega}_s$ and the worker is constrained at a wage of $\underline{\omega}_s$, $\Pi_s(\bar{\omega}_s) = 0$ and $U_s(\underline{\omega}_s) = 0$.

First, consider the case where $\underline{\omega}_2 > \bar{\omega}_1$, so the two wage intervals do not overlap. It follows from our discussion above that for any $\omega_1 \in [\underline{\omega}_1, \bar{\omega}_1]$, if state 1 occurs next period, then the wage remains fixed at ω_1 and if state 2 occurs, then the wage rises to $\underline{\omega}_2$. Similarly, for any $\omega_2 \in [\underline{\omega}_2, \bar{\omega}_2]$, if state 1 occurs next period, then the wage falls to $\bar{\omega}_1$ and if state 2 occurs, then the wage remains at ω_2. This is illustrated schematically as follows:



Using this structure, the net expected discounted profits can be written as:

$$\begin{aligned}\Pi_1(\omega_1) &= \mathrm{w}(1) - \omega_1 + \beta p_1 \Pi_1(\omega_1) + \beta p_2 \Pi_2(\underline{\omega}_2),\\ \Pi_2(\omega_2) &= \mathrm{w}(2) - \omega_2 + \beta p_1 \Pi_1(\bar{\omega}_1) + \beta p_2 \Pi_2(\omega_2).\end{aligned}$$

Setting $\omega_1 = \bar{\omega}_1$, $\omega_2 = \underline{\omega}_2$ and using $\Pi_1(\bar{\omega}_1) = 0$ and eliminating $\Pi_2(\underline{\omega}_2)$ gives

$$(1 - \beta p_2)\,(\mathrm{w}(1) - \bar{\omega}_1) + \beta p_2\,(\mathrm{w}(2) - \underline{\omega}_2) = 0. \tag{3}$$

Similarly, since $U_2(\underline{\omega}_2) = 0$, the net expected discounted utilities for the worker satisfy:

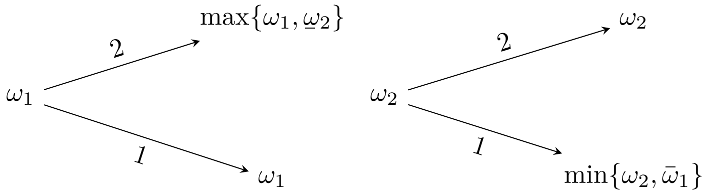
$$\begin{aligned}U_1(\bar{\omega}_1) &= u(\bar{\omega}_1) - u(\mathrm{w}(1)) + \beta p_1 U_1(\bar{\omega}_1),\\ 0 &= u(\underline{\omega}_2) - u(\mathrm{w}(2)) + \beta p_1 U_1(\bar{\omega}_1).\end{aligned}$$

Eliminating $U_1(\bar{\omega}_1)$ gives

$$\beta p_1\,(u(\bar{\omega}_1) - u(\mathrm{w}(1))) + (1 - \beta p_1)\,(u(\underline{\omega}_2) - u(\mathrm{w}(2))) = 0. \tag{4}$$

Equations (3) and (4) can be solved for $\bar{\omega}_1$ and $\underline{\omega}_2$. Note that $\bar{\omega}_1 = \mathrm{w}(1)$ and $\underline{\omega}_2 = \mathrm{w}(2)$ is a solution to these equation, but there will be another non-trivial solution if β is large enough.

Next consider the case where the two intervals overlap: $\underline{\omega}_2 < \bar{\omega}_1$. In this case the situation is even simpler. For any wage within the range where the intervals overlap, $\omega \in [\underline{\omega}_2, \bar{\omega}_1]$ the wage is kept constant next period independently of which state occurs. Therefore, for any $\omega_1 \in [\mathrm{w}(1), \bar{\omega}_1]$ and any $\omega_2 \in [\underline{\omega}_2, \mathrm{w}(2)]$ we have schematically:



Evaluating the net expected discounted profit of the firm at the wage of $\bar{\omega}_1$ and using $\Pi_1(\bar{\omega}_1) = 0$ gives:

$$\begin{aligned} 0 &= \mathrm{w}(1) - \bar{\omega}_1 + \beta p_2 \Pi_2(\bar{\omega}_1), \\ \Pi_2(\bar{\omega}_1) &= \mathrm{w}(2) - \bar{\omega}_1 + \beta p_2 \Pi_2(\bar{\omega}_1). \end{aligned}$$

Eliminating $\Pi_2(\bar{\omega}_1)$ from these two equations gives:

$$\bar{\omega}_1 = (1 - \beta p_2)\mathrm{w}(1) + \beta p_2 \mathrm{w}(2). \tag{5}$$

Similarly, evaluating the net expected discounted utility for the worker at a wage of $\underline{\omega}_2$ and using $U_2(\underline{\omega}_2) = 0$ gives:

$$\begin{aligned} U_1(\underline{\omega}_2) &= u(\underline{\omega}_2) - u(\mathrm{w}(1)) + \beta p_1 U_1(\underline{\omega}_2), \\ 0 &= u(\underline{\omega}_2) - u(\mathrm{w}(2)) + \beta p_1 U_1(\underline{\omega}_2). \end{aligned}$$

Eliminating $U_1(\underline{\omega}_2)$ from these two equations gives:

$$u(\underline{\omega}_2) = \beta p_1 u(\mathrm{w}(1)) + (1 - \beta p_1) u(\mathrm{w}(2)). \tag{6}$$

Equations (5) and (6) determine $\bar{\omega}_1$ and $\underline{\omega}_2$ respectively. Note that as $\beta \to 1$, $\bar{\omega}_1 \to \mathrm{w}^*$, the expected spot wage, and $\underline{\omega}_2 \to \mathrm{w}_*$, the certainty equivalent wage.

EXAMPLE *For $u(\omega) = \sqrt{\omega}$, $\mathrm{w}(1) = 1$, $\mathrm{w}(2) = 4$, $p_1 = p_2 = 1/2$, the intervals satisfy*

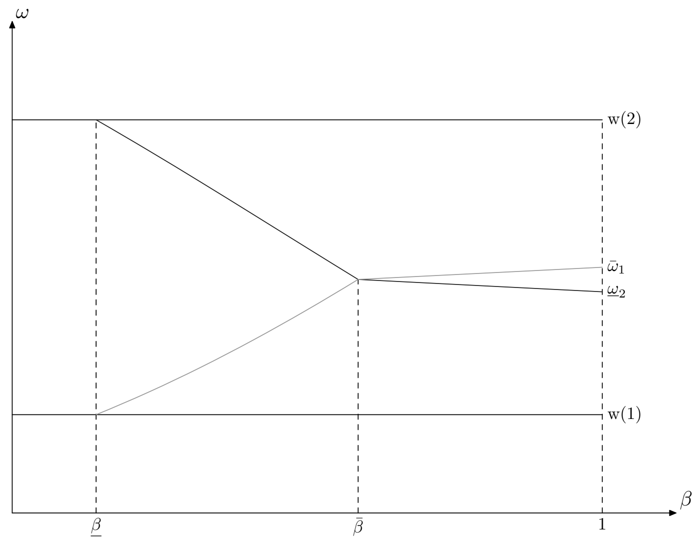
$$(\bar{\omega}_1, \underline{\omega}_2) = \begin{cases} (1, 4) & \text{for } \beta \le \underline{\beta}, \\ \left(\left(\frac{1 + \frac{\beta}{2}(1 + \frac{\beta}{2})(3 - \beta)}{1 - \frac{3}{2}\beta(1 - \frac{\beta}{2})} \right)^2, 4 \left(\frac{\frac{\beta}{2}(2 - \frac{\beta}{2}(1 - \frac{\beta}{2})) - 1}{1 - \frac{3}{2}\beta(1 - \frac{\beta}{2})} \right)^2 \right) & \text{for } \beta \in (\underline{\beta}, \bar{\beta}), \\ \left(1 + 3\frac{\beta}{2}, \left(2 - \frac{\beta}{2} \right)^2 \right) & \text{for } \beta \ge \bar{\beta}, \end{cases}$$

where $\underline{\beta} = 2\sqrt{2} - 2$ and $\bar{\beta} = 7 - \sqrt{37}$. As $\beta \to 1$, $\bar{\omega}_1 \to 5/2$ and $\underline{\omega}_2 \to 9/4$.

The interval endpoints $\bar{\omega}_1$ and $\underline{\omega}_2$ are plotted against β in Figure 2. If the firm can commit, as in Harris and Holmström (1982), then $\bar{\omega}_1 = 4$ and $\underline{\omega}_2 = (2 - (\beta/2))^2$ for all β.

Figure 2

Interval endpoints plotted against β for $u(\omega) = \sqrt{\omega}$, w(1) = 1, w(2) = 4 and $p_1 = p_2 = 1/2$.



4.3 Dynamic Programming Methodology

In this section, we describe the dynamic programming approach that uses promised utility as a state variable and imposes a promise-keeping constraint.

DEFINITION 4 *For any history (h_{t-1}, s) and a feasible value of the worker's net future utility U_s, such that a self-enforcing contract exists, the Pareto-frontier, independent of time, is given by*

Problem B $$f_s(U_s) = \sup_{\delta \in \Lambda(s)} \left\{ \Pi_t\left((h_{t-1}, s); \delta\right) \mid U_t\left((h_{t-1}, s); \delta\right) \geq \bar{U}_s \right\}.$$

We refer to $f_s(U_s)$ as the state s Pareto-frontier. The Pareto-frontier is downward sloping because a smaller value of U_s expands the constraint set and therefore, the optimum value for $f_s(U_s)$ will be larger. It can be checked that the set of self-enforcing $\Lambda(s)$ is convex and also that $f_s(U_s)$ is continuous, concave and differentiable. We refer to the value U_s as the *promised utility* for reasons that will be made clear below. It is the worker's net future utility starting from state s. Since $f_s(U_s)$ is downward sloping and continuous, the set of feasible values of the promised utility is a compact interval $I_s := [0, \bar{U}_s]$ where $f_s(\bar{U}_s) = 0$. The value $\bar{U}_s$, is the highest optimum net future utility of the worker in state s. It is independent of time. Any higher value of U_s would induce the firm to renege. Since $f_s(U_s)$ is continuous, the set of feasible values of the firm's net future utility is $[0, \bar{\Pi}_s]$ where $\bar{\Pi}_s = f_s(0)$. The intervals $[0, \bar{U}_s]$ play a key role because the contract wage is related to U_s via the Pareto-frontier. Since I_s is compact, a solution to the above problem exists and we can replace sup with max in Problem B. Moreover, given the convexity of $\Lambda(s)$ and the strict concavity of u, there will be a unique global optimum for δ. Equally it follows that $f_s(U_s)$ is strictly concave.[11]

We now show how the above problem can be treated recursively and the solution found by dynamic programming using the promised utility as a state variable. An efficient contract cannot be Pareto-dominated after any history. If it were, it would be possible to replace that part of the contract, which was dominated, simultaneously relaxing all the previous self-enforcing constraints. The new contract would then be self-enforcing and dominate the old one at date $t = 1$. Therefore, the old contract could not have been efficient in the first place. Since an efficient contract is not Pareto dominated after any history it follows that

Problem C
$$f_s(U_s) = \max_{\omega\in[a,b],(U_q)_{q=1}^S} \mathrm{w}(s) - \omega + \beta \sum_{q=1}^{S} p_q f_q(U_q),$$

subject to

$$u(\omega) - u(\mathrm{w}(s)) + \beta \sum_{q=1}^{S} p_q U_q \geq U_s, \tag{7}$$

$$U_q \geq 0 \quad \text{for all } q = 1, 2, ..., S, \tag{8}$$

$$f_q(U_q) \geq 0 \quad \text{for all } q = 1, 2, ..., S. \tag{9}$$

Constraint (7) is the promise keeping constraint that says the whatever was previously promised to the worker in terms of expected discounted future utility from a contract U_s, then the contract must deliver at least that amount in utility terms. Constraint (8) is the self-enforcing constraint for the worker and (9) is the self-enforcing constraint for the firm, which must be respected at each date and state. The range for the the choice variable U_q is the interval I_q endogenously defined by the constraints (8) and (9). The range for the wage ω is the exogenous interval $[a, b]$ (see, Assumptions 2 and 3).

The solution is found recursively by starting with $\widehat{U}_1$ and state s_1 at date $t = 1$ in Problem A. Then, problem C is solved to find the initial wage and the future promised utilities as a function of $\widehat{U}_1$ and the current state s. Depending on the state that occurs next period, the corresponding promise that was solved for in the first stage is entered into equation (7) and the maximisation problem in Problem C is re-solved for the next period, and so on.

First-order conditions

We will now show how Proposition 1 is derived from the relevant first-order conditions. Let λ_s denote the multiplier on the promise keeping constraint (7) and let $\beta p_q \mu_q$ and $\beta p_q \nu_q$ denote the multiplies on the constraints (8) and (9) in

[11]It can also be established that $f_s(U_s)$ is continuously differentiable on the interior of I_s.

Problem C. The first-order conditions are:

$$-1 + \lambda_s u'(\omega) = 0$$
$$f_q'(U_q) + \lambda_s + \mu_q + \nu_q f_q'(U_q) = 0 \quad \text{for all } q = 1, 2, ..., S.$$

Provided there exists a non-trivial contract, the multipliers μ_q and ν_q will hold with complementary slackness: both can't be positive. There is also an envelope condition

$$f_s'(U_s) = -\lambda_s.$$

Let $h(\omega) := 1/u'(\omega)$. By strict concavity of u, h is strictly increasing. Combining the equations above gives

$$f_s'(U_s) = -h(\omega) \tag{10}$$
$$h(\omega) = (1 + \nu_q)h(\omega_q) - \mu_q \quad \text{for all } q = 1, 2, ..., S, \tag{11}$$

where ω_q is the next period wage in state q. If neither of the constraints (8) and (9) bind in state q, then $\omega_q = \omega$, that is, the contract wage is unchanged from the previous period. Since $f_s(U_s)$ is concave and $h(w)$ is increasing, it follows from equation (10) that ω is positively related to U_s. Therefore, if the worker is constrained, $U_s = 0$, and correspondingly, the wage is $\underline{\omega}_s$ where $-h(\underline{\omega}_s) = f_s'(0)$. Similarly, if the firm is constrained, $-h(\bar{\omega}_s) = f_s'(\bar{U}_s)$.[12] This establishes the updating rule for the wage given in Proposition 1. The proof of the remainder of the proposition is given in the Appendix.

4.4 Two-State Example

For the two-state example, the Pareto-frontiers $f_1(U_1)$ and $f_2(U_2)$ can be computed using the same methods used to derive the interval endpoints $\bar{\omega}_1$ and $\underline{\omega}_2$ in subsection 4.2. For the case where the intervals do not overlap, $\mathrm{w}(2) \geq \underline{\omega}_2 \geq \bar{\omega}_1 \geq \mathrm{w}(1)$:

$$f_1(U_1) = \frac{\bar{\omega}_1 - u^{-1}\left((1 - \beta p_1)U_1 + u(\mathrm{w}(1))\right)}{1 - \beta p_1},$$
$$f_2(U_2) = \frac{\mathrm{w}(2) - u^{-1}\left((1 - \beta p_2)U_2 + u(\underline{\omega}_2)\right)}{1 - \beta p_2}.$$

For the case where the intervals do overlap, $\mathrm{w}(2) > \bar{\omega}_1 > \underline{\omega}_2 > \mathrm{w}(1)$:

$$f_1(U_1) = \begin{cases} \frac{\bar{\omega}_1 + \frac{\beta p_2(\bar{\omega}_1 - \underline{\omega}_2)}{1-\beta} - u^{-1}((1-\beta p_1)U_1 + u(\mathrm{w}(1)))}{1-\beta p_1} & \text{for } U_1 \in [0, U_1^c], \\ \frac{\bar{\omega}_1 - u^{-1}((1-\beta)U_1 + u(\mathrm{w}(1)) + \beta p_2(u(\mathrm{w}(2)) - u(\mathrm{w}(1))))}{1-\beta} & \text{for } U_1 \in (U_1^c, \bar{U}_1], \end{cases}$$

[12] More properly $f_s'(0)$ is the right-hand derivative at $U_s = 0$ and $f_s'(\bar{U}_s)$ is the left-hand derivative at $U_s = \bar{U}_s$.

$$f_2(U_2) = \begin{cases} \frac{\mathrm{w}(2)-\beta p_1(\mathrm{w}(2)-\mathrm{w}(1))-u^{-1}((1-\beta)U_2+u(\underline{\omega}_2))}{1-\beta} & \text{for } U_2 \in [0, U_2^c], \\ \frac{\mathrm{w}(2)-u^{-1}\left((1-\beta p_2)U_2+u(\underline{\omega}_2)-\frac{\beta p_1(u(\bar{\omega}_1)-u(\underline{\omega}_2))}{1-\beta}\right)}{1-\beta p_2} & \text{for } U_2 \in (U_2^c, \bar{U}_2], \end{cases}$$

where $U_1^c = u(\mathrm{w}(2)) - u(\mathrm{w}(1))$, $U_2^c = (u(\bar{\omega}_1) - u(\underline{\omega}_2))/(1-\beta)$. It can be checked that $f_i(U_i)$ is continuous and continuously differentiable in U_i for $i = 1, 2$. Whether the intervals overlap or not, it can also be checked that:

$$\bar{U}_1 = \frac{u(\bar{\omega}_1) - u(\mathrm{w}(1)) + \frac{\beta p_2 (u(\bar{\omega}_1)-u(\underline{\omega}_2))^+}{1-\beta}}{1-\beta p_1}; \quad f_1(0) = \frac{\bar{\omega}_1 - \mathrm{w}(1) + \frac{\beta p_2(\bar{\omega}_1-\underline{\omega}_2)^+}{1-\beta}}{1-\beta p_1};$$

$$\bar{U}_2 = \frac{u(\mathrm{w}(2)) - u(\underline{\omega}_2) + \frac{\beta p_1 (u(\bar{\omega}_1)-u(\underline{\omega}_2))^+}{1-\beta}}{1-\beta p_2}; \quad f_2(0) = \frac{\mathrm{w}(2) - \underline{\omega}_2 + \frac{\beta p_1(\bar{\omega}_1-\underline{\omega}_2)^+}{1-\beta}}{1-\beta p_2};$$

where $(x)^+ := \max\{0, x\}$. It can be checked that where the intervals overlap $\bar{U}_1 = U_1^c + U_2^c$. It can also be checked that[13]

$$\begin{aligned} f_1(U_1) &= f_2(U_1 - U_1^c) - (\mathrm{w}(2) - \mathrm{w}(1)), \\ f_2(U_2) &= f_1(U_2 + U_1^c) + (\mathrm{w}(2) - \mathrm{w}(1)). \end{aligned} \tag{12}$$

Figure 3 illustrates the Pareto-frontiers $f_1(U_1)$ and $f_2(U_2)$ in the two cases where the intervals do and do not overlap. When the two intervals do not overlap, the Pareto-frontier $f_2(U_2)$ is more steeply sloped than $f_1(U_1)$, in particular, $f_2'(0) < f_1'(\bar{U}_1)$. Whereas, when the two intervals do overlap, $f_2'(0) > f_1'(\bar{U}_1)$ and, in particular, differentiating the equations in (12) give $f_2'(U_2^c) = f_1'(\bar{U}_1)$ and $f_2'(0) = f_1'(U_1^c)$ where, from the first-order conditions, $f_1'(\bar{U}_1) = -h(\bar{\omega}_1)$ and $f_1'(U_1^c) = -h(\underline{\omega}_2)$.

Ex Ante and Ex Post Pareto-Frontiers

The Pareto-frontiers $f_1(U_1)$ and $f_2(U_2)$ are ex post Pareto-frontiers, that is, they are conditional on the state. It is possible to define an ex ante Pareto-frontier as in Kocherlakota (1996).[14] In this case, define an expected discounted utility V where $V \in [0, \bar{V}]$ with $\bar{V} = \sum_{q=1}^S \bar{U}_q$. Then, the ex ante Pareto-frontier is given by the solution to:

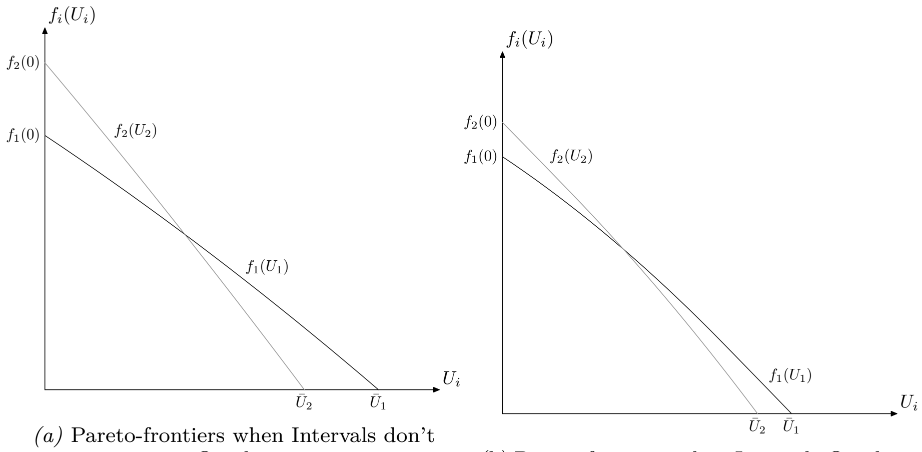
Problem D $$g(V) = \max_{(\omega_q)_{q=1}^S, (V_q)_{q=1}^S} \sum_{q=1}^S (\mathrm{w}(q) - \omega_q) + \beta \sum_{q=1}^S p_q g(V_q),$$

subject to

$$\sum_{q=1}^S p_q \left(u(\omega_q) - u(\mathrm{w}(q))\right) + \beta \sum_{q=1}^S p_q V_q \geq V,$$
$$V_q \geq 0 \quad \text{for all } q = 1, 2, ..., S,$$

[13] These follow from the definitions given in Problem C.

[14] Kocherlakota (1996) has two risk averse agents with a common utility function and a fixed aggregate income but it is easy to translate his analysis into the wage context.



(a) Pareto-frontiers when Intervals don't Overlap

(b) Pareto-frontiers when Intervals Overlap

Figure 3

Example of Pareto-Frontiers. Note that panels (a) and (b) are drawn at a different scale. The Pareto-frontiers drawn in panel (b) for a higher value of β dominate the Pareto-frontiers drawn in panel (a).

$$g(V_q) \geq 0 \quad \text{for all } q = 1, 2, ..., S,$$

where V_q is the next period expected discounted utility contingent on state q today. The relationship between Problems C and D is given by the solution of the following problem:

Problem E $$g(V) := \max_{(U_q)_{q=1}^{S}} \sum_{q=1}^{S} p_q f_q(U_q),$$

subject to

$$\sum_{q=1}^{S} p_q U_q \geq V, \tag{13}$$

$$U_q \geq 0 \quad \text{for all } q = 1, 2, ..., S, \tag{14}$$

$$f_q(U_q) \geq 0 \quad \text{for all } q = 1, 2, ..., S, \tag{15}$$

where λ^v is the multiplier on constraint (13). The solution to problem E is straightforward. The promised utility U_q is set such that $-f_q'(U_q) = \lambda^v$ for each q, unless $-f_q'(0) \geq \lambda^v$, in which case $U_q = 0$, or $-f_q'(\bar{U}_q) \leq \lambda^v$, in which case $U_q = \bar{U}_q$. The function $g(V)$ is strictly concave because it is a weighted sum of the strictly concave functions $f_q(U_q)$.

It is known that the ex post Pareto frontiers are continuously differentiable. However, although the ex ante frontier is concave, it is not necessarily differentiable. In particular, Koeppl (2006) shows that the ex ante Pareto-frontier is differentiable when all intervals overlaps but it is non-differentiable at points where the intervals do not overlap. The intuition is as follows. The expected utility V can be varied by changing the wage in one or more states. However, if the intervals do not overlap, then a marginal increase in V where the constraints are binding requires an increase in a higher wage in the higher state and a marginal decrease in V requires a decrease in the lower wage in the lower state.

We can illustrate this result in our two-state case. First consider the case where the intervals do not overlap. In this case, the solution depends on the sign of $V - p_1\bar{U}_1$. In particular, solving Problem E:

$$(U_1, U_2) = \begin{cases} \left(\frac{V}{p_1}, 0\right) & \text{for } V < p_1\bar{U}_1, \\ \left(\bar{U}_1, \frac{V - p_1\bar{U}_1}{p_2}\right) & \text{for } V \geq p_1\bar{U}_1, \end{cases}$$

and consequently,

$$g(V) = \begin{cases} p_1 f_1\left(\frac{V}{p_1}\right) + p_2 f_2(0) & \text{for } V < p_1\bar{U}_1, \\ p_2 f_2\left(\frac{V - p_1\bar{U}_1}{p_2}\right) & \text{for } V \geq p_1\bar{U}_1. \end{cases}$$

Note that $g(V)$ is continuous at $V = p_1\bar{U}_1$ since $f_1(\bar{U}_1) = 0$. However, it is not differentiable at $V = p_1\bar{U}_1$. In particular, at $V = p_1\bar{U}_1$, the left-hand derivative is $g'_-(V) = f'_1(\bar{U}_1) = -h(\bar{\omega}_1)$ and the right-hand derivative is $g'_+(V) = f'_2(0) = -h(\underline{\omega}_2)$. Since $\underline{\omega}_2 > \bar{\omega}_1$ and h is strictly increasing, $g'_+(V) < g'_-(V)$ at $V = \bar{U}_1/p_1$. The reason is that the wage is not varied continuously and jumps from $\bar{\omega}_1$ to $\underline{\omega}_2$ as V is increased in the neighbourhood of $V = \bar{U}_1/p_1$. Since the intervals do not overlap for the range $\beta \in (\underline{\beta}, \bar{\beta})$, this non-differentiability applies for a range of different parameter values.

The function $g(V)$ is however, differentiable when the intervals overlap. In this case, for $V \in (p_1U_1^c, p_1\bar{U}_1 + p_2U_2^c)$, U_1 and U_2 satisfy $f'_1(U_1) = f'_2(U_2)$, which from the definitions of the Pareto-frontiers given above, implies that $U_1 = U_2 + U_1^c$. Since $p_1U_1 + p_2U_2 = V$ it follows that:

$$(U_1, U_2) = \begin{cases} \left(\frac{V}{p_1}, 0\right) & \text{for } V \leq p_1U_1^c, \\ (V + p_2U_1^c, V - p_1U_1^c) & \text{for } V \in (p_1U_1^c, p_1\bar{U}_1 + p_2U_2^c), \\ \left(\bar{U}_1, \frac{V - p_1\bar{U}_1}{p_2}\right) & \text{for } V \geq p_1\bar{U}_1 + p_2U_2^c, \end{cases}$$

and consequently,

$$g(V) = \begin{cases} p_1 f_1\left(\frac{V}{p_1}\right) + p_2 f_2(0) & \text{for } V \leq p_1U_1^c, \\ p_1 f_1(V + p_2U_1^c) + p_2 f_2(V - p_1U_1^c) & \text{for } V \in (p_1U_1^c, p_1\bar{U}_1 + p_2U_2^c), \\ p_2 f_2\left(\frac{V - p_1\bar{U}_1}{p_2}\right) & \text{for } V \geq p_1\bar{U}_1 + p_2U_2^c. \end{cases}$$

It is easy to see that $g(V)$ is continuous and it is differentiable everywhere too. At $V = U_1^c/p_1$, the left-hand derivative is $g'_-(V) = f'_1(U_1^c)$ and the right-hand derivative is $g'_-(V) = p_1 f'_1(U_1^c) + p_2 f'_2(0) = f'_1(U_1^c)$ since $f'_2(0) = f'_1(U_1^c)$. Similarly, at $V = p_1\bar{U}_1 + p_2 U_2^c$, $g'_+(V) = f'_2(U_2^c)$ and since $\bar{U}_1 = U_1^c + U_2^c$ and $f'_1(\bar{U}_1) = f'_2(U_2^c)$, it follows that $g'_-(V) = p_1 f'_1(\bar{U}_1) + p_2 f'_2(U_2^c)$. Hence, when the intervals overlap, the function $g(V)$ is everywhere differentiable and together with concavity this establishes that $g(V)$ is continuously differentiable.

5 *Discussion*

5.1 *Renegotiation-Proofness*

Assumption 1 states that a default involves the firm offering, or worker taking, the spot market wage. That is following default there is reversion to autarky without a contract. This in not renegotiation-proof because there are Pareto-efficient allocations that dominate autarky. Nevertheless, as pointed out by Asheim and Strand (1991), the solution we have identified is strongly renegotiation-proof. The reason is that if the worker defaults in state s and their net expected gain is zero, then the efficient contract that delivers the same zero net gain to the worker offers $f_s(0)$ to the firm. That is, following a default by the worker, an efficient contract can be offered starting from the contract wage $\underline{\omega}_s$. It is like restarting the contract when the firm has all the bargaining power. Likewise, if the firm defaults, start the contract again from a wage of $\bar{\omega}_s$ where the worker has all the bargaining power and receives a net utility gain of $\bar{U}_s$.[15]

5.2 *Extensions and Applications*

The model of self-enforcing risk-sharing contracts has been applied to explain empirical evidence on departures from full insurance for workers in response to firm shocks (see, for example, Lagakos and Ordoñez, 2011; Breslin et al., 2022). It has also been extended and applied to a variety of different contexts beyond the firm-worker case. It has been applied to household bargaining (see, for example, Mazzocco, 2007; Voena, 2015); competition between political parties (see, for example, Dixit, Grossman, and Gul, 2000; Acemoglu, Golosov, and Tsyvinski, 2011); asset pricing (see, for example, Alvarez and Jermann, 2000, 2001); risk sharing in village economies (see, for example, Ligon, Thomas, and Worrall, 2002; Dubois, Jullien, and Magnac, 2008); currency and fiscal unions (see, for example, Castro and Koumtingué, 2014; Farhi and Werning, 2017; Picard and Worrall, 2020); sovereign debt (see, for example, Worrall, 1990; Martins-da-Rocha and

[15] The concept of strongly renegotiation-proof equilibria was introduced by Farrell and Maskin (1989). The idea is that any equilibria which involves an outcome that is Pareto-dominated after some history is not credible because the players will renegotiate to the Pareto-dominant outcome. The resulting equilibria are weakly renegotiation-proof. If every weakly renegotiation-proof equilibrium is regarded as credible, then players would, after any history, always negotiate toward a Pareto-dominant weakly renegotiation-proof equilibrium. The resulting equilibria are strongly renegotiation-proof.

Vailakis, 2017) and international risk-sharing (see, for example, Kehoe and Perri, 2004).

Thomas and Worrall (2007) extend the analysis to a continuum of infinitely-lived agents who are either employed or unemployed (see also, Krueger and Perri, 2011; Broer, 2013). Genicot and Ray (2003) consider multiple agents and coalition formation. Miao and Zhang (2015) extend the analysis to continuous time. Thomas and Worrall (1994) consider a model where one agent, a transnational corporation, makes an investment decision and the other agent, a host country, can expropriate the output. Thomas and Worrall (2018) extend the analysis to a case where both agents make an investment or action that determines a joint output. Ligon, Thomas, and Worrall (2000) extended the model to allow for individual savings and Ábrahám and Laczó (2018) to public storage.

5.3 Endogenous Outside Options and Relational Contracts

In the basic model outlined above, the outside option for the worker is to receive the spot wage forever. Other possibilities have been considered in subsequent literature, where the outside option is endogenously determined by a market for contracts.[16] In Sigouin (2004) a worker who quits faces a probability of unemployment until matched with a new employer. In this new match, the worker receives all of the match surplus, and it is possible that the worker might immediately transfer to a new job and receive all the surplus. If this occurred with probability one, and because all firms are subject to the same aggregate shock, no risk sharing would be possible and the wage would always equal the aggregate productivity shock (spot wage). But if the probability of finding a new match is less than one, then there is a cost of quitting and this allows some risk sharing to be supported. Likewise, Rudanko (2009) and Kudlyak (2014) consider a similar setup but in a frictional labour market with directed search. In Rudanko there is always at least one period of unemployment, and the split of surplus in a new contract is determined by an equilibrium contract that trades off promised utilities with job acceptance probabilities. The potential unemployment spell endogenously determines the degree of feasible risk sharing.[17]

These models can be solved in a similar fashion to that outlined above, and the basic characterisation is the same, with the search option replacing trading on the spot market for ever, but the solution requires solving for the endogenous outside option values as well.

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[16] It is worth mentioning the literature on cooperation without information flows (see, for example, Ghosh and Ray, 2023, this issue). In this literature, deviations from co-operation are not observable outside a relationship and parties may move to a new one without delay. In this case, the terms of the relationship must deter deviation and the only way to do this is by a gradual increase in cooperation over the duration of the relationship or by endogenously creating a barrier to entry into new relationships.

[17] In both Sigouin (2004) and Rudanko (2009) existing matches dissolve with a constant probability, providing a flow of workers looking for new contracts, and if the cause of a separation is not observable to outside parties, a new employer cannot discriminate against a worker who has reneged on a previous contract.

However, there is one new additional issue that arises. Consider the two-state example in our spot-wage model, and suppose that a contract at date t offers the highest feasible utility $\bar{U}_2$ in state 2. In this case, the wage is $\bar{\omega}_2$ and the firm has zero surplus, $\Pi_2(\bar{\omega}_2) = 0$ today. Since $\omega(h_t, s_{t+1}) = \bar{\omega}_{s_{t+1}}$, in either state tomorrow $\Pi_s(\bar{\omega}_s) = 0$ and the firm has zero surplus in the future as well. Also, suppose that the firm makes a take-it-or-leave-it wage offer each period. If the firm were to deviate from the optimal wage by offering $\bar{\omega}_2 - \varepsilon$ with $\varepsilon > 0$ and small, then the worker would assume that the firm will not honour the contract in future. Thus, the worker would be better off with the spot wage $\mathrm{w}(2)$ since $\bar{\omega}_2 = \mathrm{w}(2)$, and should quit immediately. Consequently, the firm has no incentive to deviate.

When, however, the outside option is another contract (after a potential period of unemployment) this argument no longer always works. To fix ideas, suppose that, as in Sigouin (2004), contracts start with the worker receiving all the match surplus. Suppose too that if the worker quits, then there is a potential spell of unemployment during which the worker receives unemployment benefit of $d < \mathrm{w}(1)$. Let φ be the constant probability that an unemployed worker gets a match each period. Here we interpret the spot wage $\mathrm{w}(s)$ as the output of a match between a firm and a worker (there being no spot market as such).[18] Then, again we have $\bar{\omega}_2 = \mathrm{w}(2)$. A match that starts in state 2 will keep the wage at $\mathrm{w}(2)$ until state 1 occurs. Suppose that state 1 has not occurred yet and, as above, the firm deviates at time t and offers $\bar{\omega}_2 - \varepsilon$ rather than $\bar{\omega}_2$. The worker can quit immediately and either gets a new match with probability φ and receives $\bar{\omega}_2$, or remains unemployed until next period. Alternatively, the worker can stay with the firm this period, receiving $\bar{\omega}_2 - \varepsilon$, and quit next period, in which case they will be in the same position from $t + 1$ as immediately quitting and failing to get a match at t.

Observation 1 *When the outside option is another contract in which the worker receives all the surplus, the probability that an unemployed worker immediately finds a new match is less than one, and the firm makes a take-it-or-leave-it wage offer each period, then in a stationary equilibrium the optimal contract subject only to the self-enforcing constraints is not implementable: a profitable deviation for the firm exists in some states in which it offers a wage cut of ε for some $\varepsilon > 0$.*

The formal argument is given in the Appendix, but the intuition is that because of discounting, getting a wage close to $\bar{\omega}_2$ for one period now and then searching, is better than searching and potentially getting $\bar{\omega}_2$ later. In the putative equilibrium, the firm gets discounted profits of zero in state 2 and zero profits in each continuation (by the updating rule). Thus, there is a profitable deviation for the firm from the putative equilibrium, since if the worker accepts

[18] We are assuming for simplicity that there is no exogenous separation, although the argument also goes through *mutatis mutandis* in that case, and exogenous separation is needed for the stationary equilibrium with a non-degenerate unemployed search market to exist.

the wage cut, the firm makes positive profits today, since $\bar{\omega}_2 - \varepsilon < \mathrm{w}(2)$ (and zero profits thereafter because the match ends), rather than the equilibrium profits of zero.

To account for this possibility an extra constraint should be incorporated to deter the firm from such a deviation. An alternative is to suppose that in order to produce the output, the worker exerts a non-contractible effort that can be withheld as a "punishment" for wage cuts. If this effort is costless and has sufficient impact on output, then withholding effort would be enough to deter the deviation. More realistically, there might be a cost to effort, and we discuss this next.

Costly Effort

Suppose that the worker needs to put in effort $e \in \{0, \bar{e}\}$ where $e = 0$ denotes shirking and leads to output $y_t = 0$ but imposes no cost on the worker, while $e = \bar{e}$ leads to output $y_t = \mathrm{w}_t$ but has a utility cost $c > 0$. We suppose, as in MacLeod and Malcomson (1989), that effort is observable to the firm. The timing is as follows: each period after observing the current state w_t, the firm makes a take-it-or-leave-it wage offer $\tilde{\omega}_t$, and the worker can quit and take her outside option; otherwise, the worker then chooses effort $e_t \in \{0, \bar{e}\}$ and finally, after observing e_t, the firm may pay a bonus $b_t \geq 0$. Worker utility at time t is $u(\omega_t + b_t) - c$. We assume that c is sufficiently small that $e_t = \bar{e}$ is always desirable. We also assume henceforth that if $e_t = 0$, then the match dissolves at the end of the period and the worker must search for another contract.[19]

The first observation is that without using the bonus, the optimal contract (with output always positive) derived above is not implementable. To see this consider in a two state example, $s_t = 1$ and $\omega(h_t) = \underline{\omega}_1$, so that the worker gets no surplus. The optimal contract specifies that the expected continuation surplus is zero at $t + 1$. Consequently, at the effort stage, the worker incurs the cost c but expects a zero surplus; by deviating to $e = 0$ she improves her utility by c.

Can an optimal risk-sharing contract be implemented in the sense that the contract wage $\omega(h_t)$ can be split into an upfront wage, $\tilde{\omega}$, and a bonus b, such that the worker is incentivised not to shirk, and the firm incentivised to pay the bonus? We can assert the following (an outline proof is given in the Appendix):

Observation 2 *If the intervals are non-degenerate, then for c sufficiently small the optimal contract with positive output is implementable.*

The intuition is that, provided there is sufficient surplus in the relationship, the contract can use future surplus to either incentivise effort directly, or if the contract offers the worker insufficient future surplus, the surplus will incentivise the firm to pay a sufficient bonus. A more general result of this kind with risk-neutral parties was established by MacLeod and Malcomson (1989), but

[19] This can be supported by strategies such that in future the firm offers a zero wage and the worker puts in zero effort.

in our case, the split between upfront wage and bonus may vary over time to maintain incentives.[20]

5.4 Current and Open Areas of Research

While there have been many extensions and applications of the basic model, research in this area is still expanding. Bold and Broer (2021) have used the model of Genicot and Ray (2003) on coalition formation to revisit models of risk sharing within village economies. An important ongoing area of research is to consider efficient separation. In Wang and Yang (2019) an agent receives a shock to the outside option and in Ábrahám and Laczó (2022) there is a random shock affecting the utility of a match. In both cases, the equilibrium may involve efficient or inefficient separation of partners. Observations 1 and 2 show that there remains work to be done when the outside option is itself an equilibrium contract. The infinitely-lived models considered above mostly have either two agents or a continuum of agents. An important avenue of research is to consider limited commitment in overlapping generations models. For example, Lancia, Russo, and Worrall (2022) consider an overlapping generations model with a continuum of agents but where only two agents are alive at any point in time.

6 Conclusion

This paper has offered a personal perspective on Thomas and Worrall (1988). We have presented the basic model in a way that we hope makes it accessible. We have discussed how some of the subsequent literature has developed and suggested some avenues of current and future research.

To conclude, we briefly consider how self-enforcing risk-sharing contracts interact with the incentive issues that have been studied in the relational contracting literature, which generally assumes the risk neutrality of contracting parties. Dilmé and Garrett (2020) consider an otherwise canonical relational contracting model, where effort is observable but non-contractible, without uncertainty but with a risk-averse agent. They show how the observability or otherwise of savings interacts with the optimal contract. Our discussion in Section 5.3 considered a model without saving (as we assumed here), but with uncertainty: if effort costs are low relative to the relationship surplus, the optimal risk-sharing contract can always be implemented with efficient effort levels provided bonuses are available. This is in the spirit of MacLeod and Malcomson (1989). In Thomas and Worrall (2018) a closely related question in a dynamic hold-up model is analysed. Both agents exert effort, are risk averse and are subject to shocks. In this case there is a trade-off between efficiency of actions and risk sharing. The hold-up problem creates the possibility that the action of one agent may be held inefficiently low to relax the constraint of

[20] This observation also applies in the original spot-market model, although the outside option interpretation is problematic because incentivising output in one-period relationships is not feasible under the current timing assumptions.

the other agent. This can in turn allow an improvement in risk sharing if the optimal actions would necessitate a substantial change in the marginal utility of income ratio. That is, the the optimal contract will sacrifice action efficiency in order to relax the relevant constraint. An implication is that the amnesia property (Section 4.1) no longer holds.

A related question is how limited commitment and risk-aversion interact with relational contracts when these lead to efficiency wages and unemployment (Shapiro and Stiglitz, 1984). There has been little work on this topic to the best of our knowledge. Golosov and Menzio (2020) consider a matching model with risk-averse workers, one-period contracts, and where worker effort is unobservable and needs to be incentivised by probability of match termination when (observed) output is low. In states where unemployment is higher, under Nash bargaining with risk-averse workers and convex hiring costs, the loss for the worker is high relative to that for the firm. The implication is that firms optimally load firing onto unemployment states and benefit from coordinating across states, leading to endogenous cycles. Whether a similar conclusion still holds when longer-term relational contracts are used (as in, for example, Board and Meyer-Ter-Vehn, 2015) is an open question.

Appendix

A.1 Proof of Proposition 1.

Proof It is shown in Thomas and Worrall (1988) that the functions $f_s(U_s)$ are concave and continuously differentiable and that U_s belongs to a compact interval $[0, \bar{U}_s]$. The interval characterisation follows from the first-order conditions given in equations (10) and (11) as explained in the text. In particular, equation (10) shows that there is a strictly increasing function ψ_s such that $\omega = \psi_s(U_s) = h^{-1}(-f_s'(U_s))$ with $\underline{\omega}_s = \psi_s(0)$ and $\bar{\omega}_s = \psi_s(\bar{U}_s)$. The interval characterisation then follows from equation (11).

For any two states k and s and any $U_s \in [0, \bar{U}_s]$

$$f_s(U_s) + \mathrm{w}(k) - \mathrm{w}(s) = f_k(U_s + u(\mathrm{w}(s)) - u(\mathrm{w}(k))). \tag{A1}$$

Differentiating this equation with respect to U_s and evaluating at $\bar{U}_s$ gives

$$f_s'(\bar{U}_s) = f_k'(\bar{U}_s + u(\mathrm{w}(s)) - u(\mathrm{w}(k))). \tag{A2}$$

Since $\mathrm{w}(k) - \mathrm{w}(s) > 0$, we have, from equation (A1) that

$$f_k(\bar{U}_s + u(\mathrm{w}(s)) - u(\mathrm{w}(k))) > f_s(\bar{U}_s) = 0 = f_k(\bar{U}_k).$$

Since f_k is decreasing, $\bar{U}_s + u(\mathrm{w}(s)) - u(\mathrm{w}(k)) < \bar{U}_k$. Since f_k is strictly concave, using equation (A2) gives

$$f_s'(\bar{U}_s) = f_k'(\bar{U}_s + u(\mathrm{w}(s)) - u(\mathrm{w}(k))) > f_k'(\bar{U}_k).$$

Using the definition of h given above, we have

$$h(\bar{\omega}_k) = -f_k'(\bar{U}_k) > -f_q'(\bar{U}_s) = h(\bar{\omega}_s).$$

Since h is increasing, it follows that $\bar{\omega}_k > \bar{\omega}_s$. Equally, a symmetric argument shows that $\underline{\omega}_k > \underline{\omega}_s$.

Since $f_s(\bar{U}_s) = 0$, $\bar{\omega}_s = \mathrm{w}(s) + \beta \mathbb{E} f_q(U_q)$. Since $f_q(U_q) \geq 0$, it follows from this that $\bar{\omega}_s \geq \mathrm{w}(s)$. In state S, starting from $\bar{U}_S$ where the wage is $\bar{\omega}_S$, it follows from the updating rule that $U_q = \bar{U}_q$ for all states q. Thus, $f_q(U_q) = 0$ for all q. Hence, $\bar{\omega}_S = \mathrm{w}(S)$. A symmetric argument shows that $\underline{\omega}_s \leq \mathrm{w}(s)$ and $\bar{\omega}_1 = \mathrm{w}(1)$. *Q.E.D.*

A.2 Proof of Observation 1.

PROOF Define R_s to be the utility the worker gets from searching in state s.[21] By assumption $\bar{U}_s$ is the utility if a match is made (i.e., $f_s(\bar{U}_s) = 0$). Hence,

$$R_s = \varphi \bar{U}_s + (1-\varphi)\left(u\left(d\right) + \beta \mathbb{E}_s\left[R_s\right]\right). \tag{A3}$$

The worker will accept a wage cut of ε if

$$u\left(\bar{\omega}_2 - \varepsilon\right) + \beta \mathbb{E}_s\left[R_s\right] > R_2. \tag{A4}$$

Also

$$\bar{U}_2 = u\left(\bar{\omega}_2\right) + \beta \mathbb{E}_s\left[\bar{U}_s\right] \tag{A5}$$

because $\bar{\omega}_2 > \bar{\omega}_1$ and by the updating result of Proposition 1 if either state occurs next period, then the continuation payoff is $\bar{U}_s$. Substituting for R_2 in (A4) using (A3),

$$u\left(\bar{\omega}_2 - \varepsilon\right) + \beta \mathbb{E}_s\left[R_s\right] > \varphi \bar{U}_2 + (1-\varphi)\left(u\left(d\right) + \beta \mathbb{E}_s\left[R_s\right]\right).$$

Rearranging:

$$(1-\varphi)\left(u\left(\bar{\omega}_2 - \varepsilon\right) - u\left(d\right)\right) > \varphi\left(\bar{U}_2 - u\left(\bar{\omega}_2 - \varepsilon\right)\right) - \beta \mathbb{E}_s\left[R_s\right]. \tag{A6}$$

Take expectations in (A3) and rearrange:

$$\mathbb{E}_s\left[R_s\right] = \frac{(1-\varphi)\, u\left(d\right) + \varphi \mathbb{E}_s\left[\bar{U}_s\right]}{1-\beta+\beta\varphi}.$$

Substitute back into (A6), use (A5) and rearrange:

$$\frac{(1-\beta)\left(\left(u\left(\bar{\omega}_2 - \varepsilon\right) - u\left(d\right)\right) + \beta\varphi\left(\frac{u(\bar{\omega}_2-\varepsilon)}{(1-\beta)} - \mathbb{E}_s\left[\bar{U}_s\right]\right)\right)}{1-\beta\left(1-\varphi\right)} > 0. \tag{A7}$$

[21] Recall we are assuming a stationary equilibrium, as in, e.g., Sigouin (2004), although, for simplicity, we are ignoring the possibility of exogenous separation.

Since lifetime utility from the contract involves some wages that are below $\bar{\omega}_2$ and none that are above, it follows that $u(\bar{\omega}_2)/(1-\beta) > \mathbb{E}_s[\bar{U}_s]$. Since $d < \mathrm{w}(1) < \bar{\omega}_2$, it therefore follows that (A7) holds for $\varepsilon = 0$ and by continuity also for $\varepsilon > 0$ but not too large. *Q.E.D.*

A.3 Proof of Observation 2.

PROOF Here we provide a sketch of the proof. If the firm deviates from the contract by paying a wage smaller than $\tilde{\omega}(h_t)$, the worker sets $e = 0$, the firm pays a zero bonus, and the match dissolves at the end of the period. Likewise, if the worker sets $e = 0$, the firm pays a zero bonus, and the match dissolves at the end of the period. Finally, if the firm pays a bonus smaller than $b(h_t)$, the match dissolves at the end of the period.

Consider currently being in state 1 at a wage ω in an optimal contract δ. Let U_s denote the worker's optimal net surplus relative to the outside option of searching for a new contract. Provided that the wage intervals are non-degenerate, $f_s(U_s)$ is decreasing on the non-degenerate interval $[0, \bar{U}_q]$. Let the continuation values as a function of ω, $\sum_q p_q U_q$ and $\sum_q p_q f(U_q)$ (*cf.* Problem C), be denoted by $\tilde{V}(\omega)$ and $\tilde{\Pi}(\omega)$ respectively. By Proposition 1, $\tilde{V}(\omega)$ is increasing in ω and and $\tilde{\Pi}(\omega)$ is decreasing. To implement the contract with effort incentivised, the wage ω is split between a pre-effort wage $\tilde{\omega}$ and a bonus b contingent on $e = \bar{e}$. The worker will exert effort if:

$$u(\omega) - c + \beta \tilde{V}(\omega) \geq u(\tilde{\omega}).$$

The left-hand side is what the worker gets including the current bonus and the future continuation whereas, if the worker shirks, she consumes the wage $\tilde{\omega}$ but receives no bonus and the contract terminates, so she loses the future surplus $\beta\tilde{V}(\omega)$. The firm will pay b if:

$$b \leq \beta \tilde{\Pi}(\omega).$$

Choose b such that $u(\bar{\omega}_1) - u(\bar{\omega}_1 - b) = c$. Then, by concavity, $u(\omega) - u(\omega - b) \geq c$ for all $\omega \in [\underline{\omega}_1, \bar{\omega}_1]$. For c small enough, there is an $\omega^* > \underline{\omega}_1$ such that $\beta\tilde{\Pi}(\omega^*) \geq b$. Therefore, for all $\underline{\omega}_1 \leq \omega \leq \omega^*$, there is sufficient surplus to incentivise the firm paying a large enough bonus to induce $\bar{e}$. Likewise, for c small enough there is an $\omega^{**} < \bar{\omega}_1$ such that $c \leq \beta\tilde{V}(\omega^{**})$, so that for all $\bar{\omega}_1 \geq \omega > \omega^{**}$ there is sufficient future surplus that the worker exerts $\bar{e}$ without a bonus. Finally, if c is small enough, then $\omega^* \geq \omega^{**}$. This implies that for any $\omega \in [\underline{\omega}_1, \bar{\omega}_1]$, either a sufficient bonus can be incentivised, or future surplus is sufficient, to elicit $\bar{e}$, so ω can always be split between $\tilde{\omega}$ and b to elicit $\bar{e}$. A similar argument applies in other states. *Q.E.D.*

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